EFFECTS OF VISUAL NOISE

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Summary

Random fluctuation in luminance, over time or space or both, is luminance noise. Squared threshold contrast is found to be proportional to the spectral density of luminance noise, plus a constant. The constant is an estimate of the observer's "equivalent noise". The observer's near-threshold performance at detection and discrimination (and possibly apparent-contrast matches) is consistent with a description of the observer as noise-free except for the equivalent noise at the visual field.

Effects of varying noise spectrum in all three spatio-temporal-frequency dimensions support the hypothesis that we detect a grating by means of a linear spatio-temporal filter, a "channel", so that squared threshold contrast is proportional to the total noise power (including equivalent noise) passed by the linear filter.

A hypothesis of intrinsic uncertainty leads to the channel-uncertainty model, which is consistent with the noise effects and predicts summation and facilitation effects as accurately as accurately as other theories which do not explain the effects of noise. The uncertainty hypothesis was confirmed by direct test: threshold contrast for one of ten-thousand possible signals was only a factor of 1.5 higher than threshold contrast of a fixed signal.

PREFACE AND ACKNOWLEDGEMENTS

The temporal-tuning data reported in Chapter 3 were collected in collaboration with A.B. Watson, with permission from the Board of Graduate Studies. Otherwise, this dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration.

Fergus W. Campbell, my PhD. supervisor, often disagreed with me, was usually right, but always let me find my own way. His keen curiousity and talent for dramatic demonstration of new understanding showed me how exciting research could be.

John G. Robson always seemed to know how an experiment would turn out, as though he had already done it years ago. He showed me how to fix a computer, design a video display, and how to approach any problem with the conviction that it can be solved without a degree in the specialty.

Gordon Legge, Dan Kersten, Gary Rubin, and Mary Schleske patiently listened again and again to my muddled explanations of what Rose meant by the "quantum efficiency of the eye", as I gradually determined how it was related to Barlow's quantum efficiency of the observer. They, Jamie Radner, and John Foley also provided helpful criticisms of my earlier drafts. I am grateful to Dan Kersten for acting as an observer in the experiment reported in Appendix 6.

My examiners, John G. Robson and Jacob Nachmias, made a number of important suggestions at my oral exam. In particular, I had not realized that channel-uncertainty is a form of probability summation.

I am grateful to the members and visitors of the Craik Laboratory for the stimulating environment they provided. I thank Clive Hood for his invaluable assistance. I am grateful to Robert Hess for many fruitful discussions on noise masking and for acting as observer in several of the experiments reported here. I am grateful to my friends A. "Beau" Watson, Dave C. Burr, and Andrew Dean for fruitful discussions, and collaborations on related projects. Roy Patterson of the MRC Applied Psychology Unit provided much helpful advice on off-frequency looking experiments, and helped me realize how worthwhile it might be to determine the predictions of the channel-uncertainty model. Correspondence with Arthur E. Burgess helped me to refine the ideas on efficiency.

Except for Appendix 6, all the experimental work was done at Cambridge University with support from a Ministry of Defense contract to F.W. Campbell entitled, "Spatial noise spectra and target detection/recognition". The writing, as well as the uncertainty experiment reported in Appendix 6, was done at University of Minnesota, where I was a Research Fellow and supported by Public Health Service Grant EY002934 to Gordon Legge.

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CHAPTER 1

OVERVIEW AND CONCLUSIONS

OVERVIEW AND CONCLUSIONS

This chapter previews the important conclusions of the thesis, and develops the notion of an observer's equivalent input noise. The observer's equivalent input noise plays a key role in the interpretation of the empirical findings.

Noise is an implicit part of most theories of visual detection, but little is known of the nature of intrinsic noise, and little attention has been paid to the studies of the effects of external noise. This thesis will consider the effects of visual noise. The approach is analogous to the standard engineering practice of describing, for example, an actual electronic amplifier as a "black box" which is equivalent to an ideal noise-free amplifier with an equivalent noise source added at its input. This is called, "referring the amplifier's noise to its input". The linearity of electronic amplifiers guarantees that all the noise in an amplifier may be referred to its input. Electronics manufacturers routinely publish the equivalent input noise spectra of their products. Designers can directly compare the signals they want to use as inputs with the equivalent input noise of the amplifier and determine several kinds of signal-to-noise ratio. The equivalent input noise usually determines the smallest signal the system can detect.

Luminance noise is random fluctuation in luminance over time or space, or both. An observer's visual system has intrinisic sources of noise, including that inherent in the probabilistic nature of light. This thesis will ask whether the observer's intrinsic noise can be referred to his input; that is, whether the observer acts as though he were noise-free except for an imputed noise at his visual field. Historically the equivalent noise has been described as a "dark light" which adds to the luminance function of the stimulus to prduce the effective stimulus; here the equivalent noise will be described as a contrast function which adds to the contrast function of the stimulus to produce the effective stimulus.

Dark light

Hecht, Schlaer, and Pirenne (1942) demonstrated that observers could detect small brief flashes which sent an average of about 100 photons into the eye. The observers' frequency-of-seeing curves were the same as that of an ideal observer (looking through a neutral density filter) whose criterion for responding "seen" was about 6 photons. As Barlow (1956) later noted, 6 photons is only a lower bound on the number of photons the observers actually utilized; any other source of variability could have limited the steepness of the frequency of seeing curves.

Barlow (1956) went on to hypothesize a "dark light":

> "One must accept the conclusion that the absorption of a single quantum <u>can</u> excite a rod, and, . . . , one must also accept the conclusion that a threshold flash which is seen <u>does</u> cause two or more rods to be excited. Apparently, the limit to the sensitivity of vision does not lie in the rods but at a later point in the nervous pathway leading to consciousness. It would be surprising if there were no explanation for the failure to detect a single excited rod, and the performance of light-sensitive physical instruments may provide a clue; here it is not the difficulty of amplifying weak signals that limits the sensitivity, but the difficulty of distinguishing a weak signal from the background of spurious signals, or 'noise,' which occurs without any light signal at all.
>
> " . . . [Noise could enter] the system anywhere between the stimulus and the final response, but if this noise is to set a limit to sensitivity it must enter the system before the level at which the threshold decision is made: this is the possibility which is pursued in this paper. Since the level of the threshold decision is not known, there is a strong case for considering first the effects of noise acting at the earliest possible point in the system - that is, to consider the effects of the rhodopsin molecule undergoing spontaneously the same reaction that occurs when it absorbs a quantum of light.

> "The assumption to be examined is that events occur in the retina, or later in the pathway, which cannot be distinguised from the events which occur when light falls on the rods and a quantum is absorbed."

Barlow (1957) decided to treat this noise as a "dark light". We will shortly take a different approach, but his analysis is instructive.

> "The first step in the quantitative approach is to justify the choice of a unit to measure the retinal noise. Following the idea that it is related to Fechner's 'Augenschwarz' or the dark light of the eye, it is here expressed in units of light intensity. Thus if it is said that the dark light has a certain value, this means that a light of this intensity entering the eye would by itself cause the same amount of noise in the visual system that it is deduced to have in total darkness. This unit is derived from the experimental measurements very directly, which is a strong reasons for choosing it . . . "

Since effective light level is the sum of the dark light and any actual light, it is expected that the actual light would be insignificant when it is much less than the dark light and dominant when it is much greater than the dark light. Since the data conform to this expectation it is reasonable to take the light level at which the transition occurs as an estimate of the dark light.

> " . . . to a first approximation, the [threshold] curves for different types of stimulus start to turn upwards at the same value of background intensity. They differ in the slope of their rising portions, and a general approximate law seems to be that the increment threshold varies as some power of the background intensity; this power varies between 0.5 and 1, but stays constant for any set of conditions over a moderate range just above threshold. Such a simple power law is represented by a straight line on the plot of log ΔI against log I, and the noise level is very easily found by the intersection of this line with the ordinate corresponding to the absolute intensity which is equivalent to the dark light."

The notion of a dark light seems to be very helpful in understanding thresholds in the dark-adapted eye, but does not readily lend itself to understanding thresholds at higher light levels. Conversely, the new description of the equivalent noise as a contrast function, now to be introduced, seems to be very helpful at non-zero light levels, but is undefined in the dark. In the future it may be found fruitful to allow both types of equivalent noise, extending the applicability of both ideas, but this thesis will only consider the equivalent noise as a contrast function.

Equivalent noise as a contrast function

Chapter 2 will review thresholds in luminance noise with a white (i.e. flat), spatio-temporal-frequency spectrum. Many authors have determined threshold contrasts for various patterns over a range of levels of white noise. All report that for each pattern the square of the threshold contrast is proportional to the sum of the spectral density of the noise mask and a constant. The constant will be called the critical spectral density. Zero-noise threshold will refer to the threshold contrast on a uniform field. The critical spectral density is the spectral density at which the resulting squared threshold is twice the squared zero-noise threshold. In this way one can determine the critical spectral density of a white noise mask for any test pattern. Figure 2.1 is a typical graph of threshold contrast versus noise level*. Both scales are logarithmic, but the threshold contrast scale is expanded so that one log unit of squared contrast (i.e. half a log unit of contrast) has the same length as one log unit of noise level. The threshold curve is flat for noise levels less than the critical noise level, and rises with unit slope (indicating proportionality of squared threshold to noise level) for noise levels greater than the critical noise level.

Chapter 2 will show that the critical spectral density is an estimate of the observer's equivalent noise. Since the squared threshold is proportional

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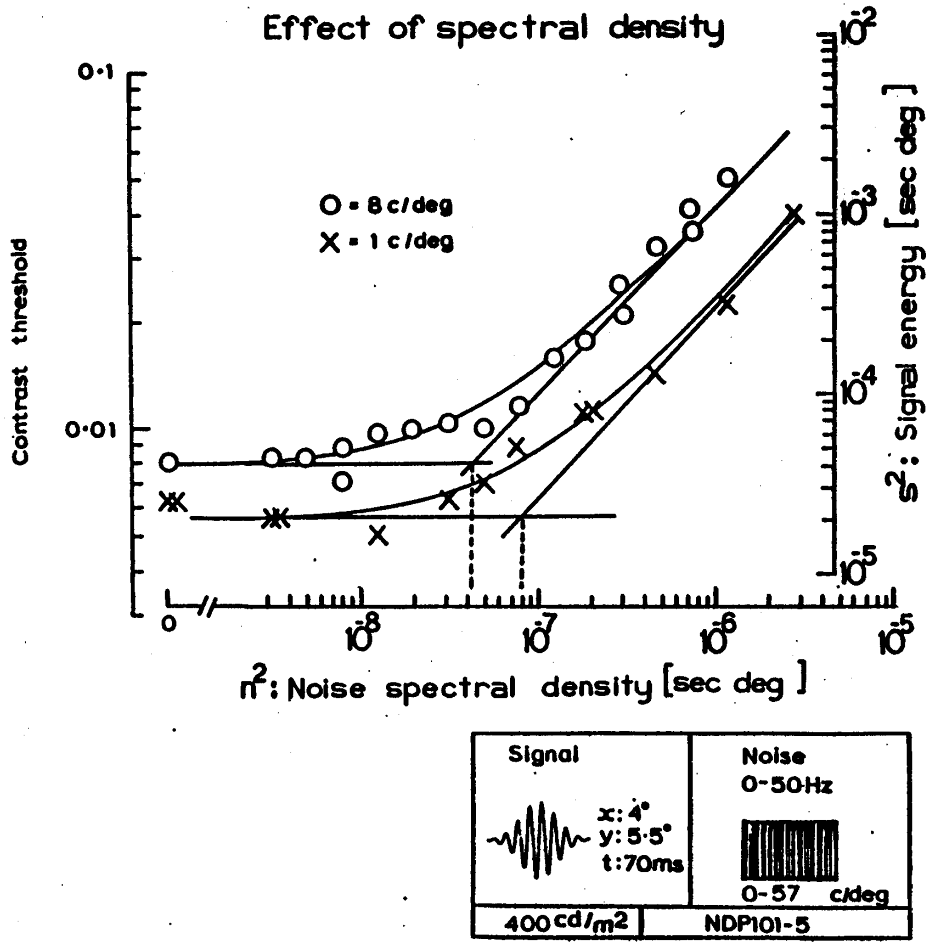
* Noise level will be used synonymously with "spectral density" of noise.



Effect of spectral density
0·1
0·01
Contrast threshold
O = 8 c/deg
X = 1 c/deg
0
10^{-8}
10^{-7}
10^{-6}
10^{-5}
n^2: Noise spectral density [sec deg]
10^{-2}
10^{-3}
10^{-4}
10^{-5}
s^2: Signal energy [sec deg]
Signal
x: 4°
y: 5·5°
t: 70ms
Noise
0-50Hz
0-57 c/deg
400 cd/m2
NDP101-5

Figure 2.1

to the sum of the actual and critical noise levels it seems appropriate to call the sum of actual and critical, the effective noise level. Even more useful is the concept of the effective stimulus, defined as the sum of the contrast function of the observer's equivalent noise and the contrast function of the stimulus. The actual stimulus is a luminance function, and can be described by a contrast function, but still has an associated luminance with a corresponding level of photon noise. The effective stimulus has no luminance; it is solely a contrast function, though it does include the observer's equivalent noise, some part of which is due to photon noise.

The effective stimulus will allow us to partition the observer into two parts, without making any assumptions about his internal structure. The first part, called "transduction" transforms the actual stimulus to the effective stimulus. The second part, "calculation", transforms the effective stimulus into the observer's performance.

Much of Chapter 2 is devoted to establishing the conditions necessary to satisfy the assumptions of Theory of Signal Detectability, and thereby the concepts of signal-to-noise ratio and efficiency. Efficiency of any transformation is the squared ratio of the output signal-to-noise ratio to the input signal-to-noise ratio. The theory of Signal Detectability guarantees that efficiency cannot exceed 1. In the absence of luminance noise, the overall efficiency of the observer, describing the efficiency of the transformation from stimulus to the observer's performance, is Barlow's "quantum efficiency". In the absence of luminance noise, "transduction efficiency" is the efficiency of the transformation from stimulus to effective stimulus. "Calculation efficiency" is the efficiency of the transformation from effective stimulus to the observer's performance. Quantum efficiency equals the product of transduction efficiency and the calculation efficiency. All three are measureable.

Chapter 3 will review thresholds in non-white noise. These data are evidence for spatial-frequency channels. The threshold of an observer detecting a sinusoidal grating is affected only by frequencies in the vicinity of the test frequency. Detection is only hindered by noise within a band of spatio-temporal frequencies about the signal frequency. This is consistent with the established idea that detection is mediated by frequency selective mechanisms, channels. The chapter will examine the hypotheses that: 1. when the observer is using a given channel, squared threshold is proportional to the total noise power passed by a linear filter; 2. the signal determines which channel is used, regardless of the noise spectrum. Chapter 3 will present evidence in support of the first notion (squared threshold proportional to total filtered-noise power), and some evidence that the second notion (one-signal-one-channel) is not always true. It seems that the observer learned to look off-frequency: using whichever channel gave the best signal-to-noise ratio, and thus the lowest threshold.

Fortunately the channel tuning functions are more or less symmetric about the test frequency so the observer would not be expected to look off-frequency when white-noise masks are used. By restricting ourselves to white noise masks we may assume the observer always detects each grating by the same channel.

Chapter 3 presents much evidence in support of its hypothesis that for any one channel, the squared threshold is proportional to total filtered noise power. This is analogous to Rushton's principle of univariance: the photoreceptor response is selective by virtue of its absorption spectrum but its response depends only on the number of photons absorbed, regardless of their spectra. In the same way the channel is selective by virtue of its filter and its threshold seems to depend only on the noise power passed by the filter, regardless of the noise spectrum.

Additive intrinsic noise may occur at the input or output or within the channel's filter. Because of the channel's univariance, threshold will depend only on the resulting noise power at the filter output. Any luminance noise which produces the same noise power at the filter output would have the same effect on threshold as these intrinsic noises. Since the filter is linear, a noise mask which doubles the squared threshold relative to the squared zero-noise threshold also doubles the noise power at the filter output, and the mask's contribution to the noise power filter output will be equal to that of the intrinsic noise. Any noise spectrum could be used, but it is convenient to use white noise. It was suggested above that the spectral density of white noise which doubles the squared threshold be called the critical spectral density. White noise at the critical spectral density is an equivalent input noise for that channel.

Of course intrinsic noise could instead be non-additive, or occur after some nonlinear transformation of the channel filter output. In general such noise could not be referred to the visual field; there may be no equivalent input noise. Chapter 4 will present evidence from contrast discrimination for such non-referable intrinsic noise, but it seems to be significant only well above threshold. Threshold, and contrast discrimination near threshold, seem to be determined by intrinsic noise that can be referred to the visual field.

Now consider all the channels taken together, that is, the observer. So far we have considered equivalent input noises for each channel, but they may differ. Can one noise spectrum be found which serves as an equivalent input noise for all channels? If so, the performance of an observer detecting patterns on a blank or noisy background could be accurately modeled by a hypothetical noise-free observer with an equivalent noise at its field of view.

The problem is to find a spectrum for the equivalent noise that produces in each channel the same variance that that channel's intrinsic noise produces. Here is an example of why the existence of an equivalent input noise for each channel does not imply the existence of an equivalent input noise for the ensemble. Suppose two channels, A and B, are such that there is no frequency to which both respond, and imagine a third channel, C, whose filter is the sum of the filters of channels A and B. Finally, suppose that channels A, B, and C have the same intrinsic variance at their filter outputs. Any luminance noise that produces the same variance in A and B will produce twice that variance in C, thus there is no noise spectrum which could serve as an equivalent input noise for all three.

However such incompatibilities may not occur, so that an equivalent input spectrum could be found for the ensemble of all channels. Then, in principle, we could synthesize luminance noise with that spectrum. This noise should raise the variance in every channel by the same proportion and therefore should raise the threshold contrasts for all patterns by the same proportion. This would be evidence that this is indeed an equivalent input noise for the observer. Unfortunately it is exceedingly difficult to synthesize noise with an arbitrary spatio-temporal spectrum.

Chapter 4 will examine effects of white noise on matches of apparent contrast and on contrast discrimination. The contrast of an unmasked <u>match grating</u> was adjusted to match the apparent contrast of a <u>test grating</u> in noise. At all noise levels, the match contrast was identical to the test contrast when both were above their thresholds, but match contrast fell sharply to its threshold when the test grating approached its threshold.

The results of contrast discrimination indicate that suprathreshold contrast discrimination is limited by a non-referable intrinsic noise. For suprathreshold contrasts the minimum discriminable contrast difference increases with test contrast, and is independent of effective noise level. Since the effective noise accounts for all referable noise, and suprathreshold contrast discrimination is independent of effective noise it must be limited by a non-referable intrinsic noise (or not limited by noise at all).

The utility of referring noise to the visual field is further enhanced by evidence that near-threshold phenomena are scale-invariant. We have already seen that the squared threshold is proportional to the effective noise level. Chapter 5 presents evidence that for detection of a grating in noise the psychometric function relating detectability to contrast is independent of noise level if contrast is plotted in units of threshold contrast. Chapter 5 also replots results of Chapter 4 to show that for near-threshold test contrasts, the relation of minimal detectable contrast difference to test contrast is independent of noise level (when both contrasts are plotted in units of threshold contrast). Near-threshold detectability and discriminability depend only on the ratio of squared contrast to the effective spectral density of the noise.

All the near-threshold phenomena reported here scale with the effective noise level. Conversely, the suprathreshold phenomena reported (contrast discrimination and apparent-contrast matches) are independent of the noise level. The effective noise level seems to determine the limits of visibility, with little or no effect above threshold. The equivalent-noise idea unifies the explanation of near-threshold performance at all noise levels. Suprathreshold phenomena will require separate explanation.

There are many models of the visual detection process that succeed in relating one aspect of visual detection to another. From the steepness of the psychometric function the "probability summation" model predicts summation effects (whereby extending a pattern, or combining disparate patterns increases detectability slightly). From the steepness of the psychometric function the "nonlinear transducer" model predicts the facilitation observed in near-threshold contrast discrimination. Neither model offers explanation for the success of the equivalent noise description.

This thesis will present evidence that: 1. the noise in a channel relevant to threshold can be referred to its input, and 2. the operation of the channel scales with the effective noise level. These are severe constraints on models of visual detection. For example, proponents of the standard version of "probability summation" (which incorporates the "high-threshold" assumption) will have to show how its intrinsic noise could be referred to the visual field. Since in this model the intrinsic noise comes from the random failure of channels to "detect", it is not clear how this could be done.

Many explanations have been offered for the steepness of the psychometric function. One of these, the "uncertainty hypothesis", is consistent with the equivalent noise result. Chapter 5 will show that, with some simplifying assumptions, this hypothesis implies that <u>channel uncertainty</u> should be an accurate model of human performance. The model is based on the way an ideal observer would detect one of many possible signals. The noise in this model may be referred to its input, and the operation of the model scales with the effective noise level. Furthermore it will emerge that this model is both a "probability summation" model (though without the high-threshold assumption) and (as has been pointed out in the past) a "nonlinear transducer" model. Calculations will be presented that show that this channel-uncertainty model gives accurate, quantitative account of all the aspects of visual detection mentioned above.

Finally, Appendix 6 (discussed in Chapter 5) will present an experiment designed to test the uncertainty hypothesis: that the observer fails to make full use of prior information about the identity of the signal. It was found that the threshold contrast for a signal (a brief thin line) which could appear at any one of ten thousand possible times and places was only a factor of 1.5 higher than when the observer knew when and where it would be presented. The small increase in threshold indicates that the observer benefits little from the knowledge of time and place, confirming the uncertainty hypothesis.

CHAPTER 2

THRESHOLDS IN WHITE NOISE

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ABSTRACT

Chapter 1 introduced the notion of the observer's equivalent noise. This chapter will show that study of the limitations of the human detection process is profitably divided into two parts. Firstly, what is the observer's equivalent noise level and what does it depend on? Secondly, given the effective noise level (the sum of the equivalent noise and any actual luminance noise), how well does the observer perform? The first question is addressed by the transduction efficiency, defined as the ratio of the photon noise to the observer's equivalent noise, or equivalently, the fraction of the corneal photons required to account for the observer's equivalent noise. Rose (1948) called it the "quantum efficiency of the eye". The second question is addressed by the calculation efficiency, defined as the observer's detective efficiency with respect to the "effective" signal-to-noise ratio. Barlow (1977) called it "central efficiency".

It is shown that (detective) quantum efficiency, QE, is the product of transduction efficiency, TE, and calculation efficiency, CE:

$$QE = TE \times CE.$$

By a brilliant argument Rose (1948) concluded that our transduction efficiency ("quantum efficiency of the eye") is between .5% and 5% for a wide range of patterns and luminances. However in order to reach this conclusion, he assumed that the calculation efficiency was constant. Constancy of both transduction and calculation efficiencies implies constancy of quantum efficiency, i.e. the Rose-de Vries law, which was subsequently shown to be violated nearly everywhere (Barlow 1958). Assumption-free estimates of transduction efficiency made here from both published and new data are all between .4% and 3%. They span nearly 7 decades of light level, gratings of 2 to 15 c/deg, discs of 5.7' to 32', and peripheral (7° nasal field) as well as foveal viewing. By implication, the failure of the Rose-de Vries law is due to changes of calculation efficiency, not transduction efficiency.

It is not generally appreciated that Rose (1946, 1948) introduced not one, but two fundamental physical measures of light sensitivity. The first, quantum efficiency, which Rose applied to tv cameras and suggested applying to photographic film, has been widely applied to all image sensors including the human observer. Rose's second measure, which he called the "quantum efficiency of the eye" is here defined in a more general way and renamed "transduction efficiency". Like quantum efficiency it is a property of anything that is light-sensitive. It should be useful in sorting out the various causes of inefficiency in all complex light sensors, including the human observer, visually responsive cells, and photographic film.

INTRODUCTION

This chapter will examine the effect of "white" luminance noise on the contrast threshold. Report of new measurements will complement the first thorough review of the existing literature. This chapter attempts to be self-contained, but the reader may wish to refer occasionally to Appendix 1, which presents a fuller explanation of the terms used to describe visual noise. Luminance noise is random fluctuations in luminance over space or time or both. Contrast threshold refers to the amplitude $(L_{max}-L_{min})/(2L_{mean})$[1] of a pattern which the observer can just detect well enough to satisfy some criterion of response.

Only "white" noise will be considered. Mathematicians sometimes define a "white" noise process as having a constant spectral density at all frequencies, but such a thing is physically impossible; infinite power would be required to produce a non-infinitesimal spectral density over an infinite bandwidth. In practice one studies mechanisms which are sensitive over only a finite bandwidth, and there is no need to create the frequency components to which there is no sensitivity. White noise here means that on each presentation there was luminance noise whose frequency components over the bandwidth of the mechanism under study, to a good approximation, had uncorrelated amplitudes with equal variance and zero mean. As will be discussed in the following pages, the goodness of the approximation was much better in some experiments, and depends greatly on how one estimates the bandwidth of the mechanism under study.

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[1] Since noise may be present, the L_{max} and L_{min} that appear in this definition of signal contrast refer to the maximum and minimum of the mathematically-expected luminance function on the signal-and-noise presentation.

There are three reasons why the bulk of the thresholds-in-noise studies used white noise: (1) theoretical convenience - several useful theorems in the theory of signal detectability assume the noise is white; (2) as a simulation of photon noise, or the actual noise encountered in image intensifiers or television; and (3) technical limitations. It is fairly easy to produce one- or two-dimensional dynamic noise which is white (over some bandwidth). Other spectra are much more difficult to achieve.

Figure 2.1 shows data I've collected which are representative of the material to be reviewed later. Contrast threshold, c, (82% correct in 2IFC) is plotted against n^2, the spectral density of the noise, for two different test patterns (sinusoidal gratings of 1 and 8 c/deg). The smooth curves drawn through the data points represent the relation:

$$c^2 = a(b+n^2).$$

This relation has been found by everyone who measured contrast thresholds in white noise. This chapter will review the evidence (in the Results and Review sections) after considering the theoretical significance of the constants a and b (in this section).

The constant of proportionality, a, will be neglected for the moment. From here on the constant b will be called the <u>critical spectral density</u>, $n^2_{critical}$, so we may rewrite our relation as,

$$c^2 \propto n^2_{critical} + n^2.$$

The solid straight lines in Figure 2.1 represent the asymptotes of this function. The horizontal line is the asymptote for noise levels approaching zero; the unity-slope line is the asymptote for noise levels approaching

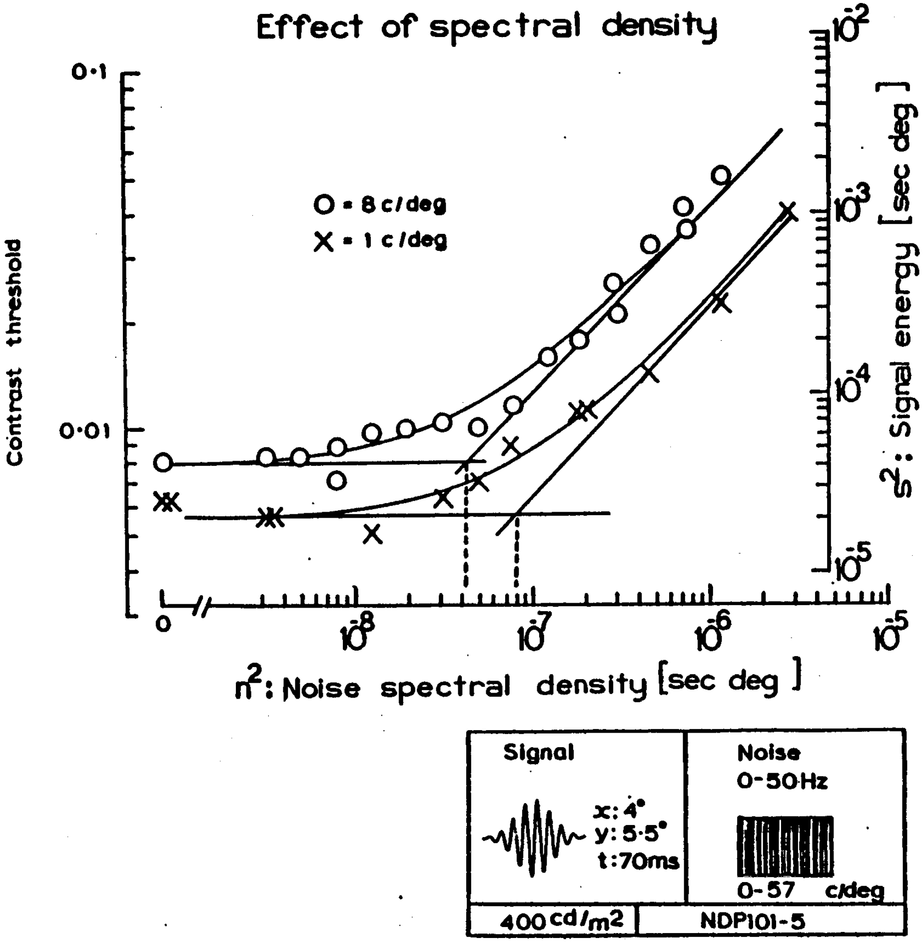
Effect of spectral density
O = 8 c/deg
X = 1 c/deg
Contrast threshold
0·1
0·01
s^2: Signal energy [sec deg]
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
10^{-8}
10^{-7}
10^{-6}
10^{-5}
n^2: Noise spectral density [sec deg]
Signal
x: 4°
y: 5·5°
t: 70ms
Noise
0-50Hz
0-57 c/deg
400 cd/m2
NDP101-5

Figure 2.1

infinity. The lines intersect at the critical spectral density, $n^2_{critical}$, indicated by the dashed lines in the figure. The "critical" spectral density is so called because it is the noise level above which threshold is primarily determined by the luminance noise, n^2, and below which threshold is primarily determined by the constant, $n^2_{critical}$.

In reviewing previous work we will need to consider two very different techniques for generating the signal and noise. In experiments with continuous displays, including mine, the signal-and-noise presentations were created by adding a signal to what would have been a noise-only presentation, but this was not true of the "dot" experiments. In the experiments using dot displays the pattern was created by modulating the probability over space and/or time of a Poisson source of identical small static or brief dots of light. Many dot experiments were motivated by the need for empirical study of vision through an image intensifier. Rose (1946, 1948) used a dot display as a simulation of photon noise. van Meeteren and Boogaard (1973) gave a good description of the appearance of such a dot, or "speck" display,

> "Once detected by the photocathode of electro-optical instruments, photons can be made visible as bright specks on an image screen by electronic amplification. When the detected photon flux is sufficiently high these specks combine in space and time to form a normal, smooth image. At lower light levels in object space, however, the detected photon flux can be so small, that the corresponding specks are visible as such, especially when the amplification is high."

Others seem to have been motivated by the premise that dots "bypass early levels of processing". Barlow (1978) discussed,

> "the demonstration by French (1954), Julesz (1964), Uttal (1969) and others that patterns composed largely of random dots provide an opportunity to probe intermediate levels of processing in the visual system. The dots which compose the pattern are, it is suggested, reliably transduced and transmitted by the lower levels, and the limitations of performance result from the ways in which the nervous system can combine and compare groups of these dots at the next higher level."

Even so, except for the special case of countably few dots, available evidence indicates that the effects of visual noise depend only on the noise spectrum, and not on whether it is the result of dots or continuous luminance fluctuation. By this interpretation, rather than bypassing anything, the dot displays are simply sufficiently noisy to dwarf the noise sources that normally limit detection.

Defining signal, noise, and the signal-to-noise ratio[2]

In analyzing thresholds in noise we need to distinguish two cases corresponding to the continuous and dot displays. In the continuous case the signal was added to an independent noise (i.e. a continuous display). In the Poisson case the signal modulated the prior probability (over space and/or time) of random dots (i.e. a dot display) or photons (i.e. a noise-free display). The continuous noise is characterized by its spectral density, $n^2(f)$, and the Poisson noise is characterized by the event rate, J_{dots}, or $J_{photons}$. Event rate will be used as a general term to refer to the density of static dots (in dots per deg^2) or flux of dynamic dots or "specks" (in dots per deg^2 sec) or photon flux (i.e. the retinal illuminance, in photons per deg^2 sec). Because of this dichotomy there has been a tendency to view continuous and dot experiments as very different. By applying a uniform terminology to both sorts of experiments it will be seen that the results are very similar, except in the few cases of countably few dots, discussed at the end of this chapter. Appendix 1 shows that the spectral density n^2 of Poisson events is

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[2] The rest of this introduction is devoted to establishing the vocabulary to be used in the Results and Review sections. This section complements Appendix 1; the molecules defined here are made up of the atoms defined there. The reader may prefer to skip directly to the Results section and refer back to here and to Appendix 1 as needed. To make this easier all defining instances are underlined.

the reciprocal of the event rate, J,

$$n^2 = 1/J.$$

The Results section will show that continuous and dot experiments produce similar graphs of threshold versus noise spectral density.

We need to establish a vocabulary which can be applied to both the continuous and Poisson cases. The experiments all sought some measure of the signal strength necessary for the observer to distinguish _noise-only presentations_, which were purely random, from _signal-and-noise presentations_, where some simple known pattern (the "signal") was present amidst the randomness.

Each noise-only presentation has a luminance function $\underline{L_N(x,y,t)}$ randomly drawn from the noise-only ensemble of possibilities, and each signal-and-noise presentation has a luminance function $\underline{L_{SN}(x,y,t)}$ randomly drawn from the signal-and-noise ensemble of possibilities. Following Linfoot (1964) define a _contrast function_ as the luminance function divided by the mean luminance, minus one. The contrast functions of the two types of presentation are

$$c_N(x,y,t) = \frac{L_N(x,y,t)}{L_{mean}} - 1$$

and

$$c_{SN}(x,y,t) = \frac{L_{SN}(x,y,t)}{L_{mean}} - 1,$$

where $\underline{L_{mean}}$ is the mean luminance on the noise-only presentations. Define $\underline{noise(x,y,t)}$ as the contrast function of the noise-only presentation:

$$noise(x,y,t) = c_N(x,y,t).$$

The contrast function of the noise, noise(x,y,t), is random, different on every presentation. Define the signal function signal(x,y,t) as the difference between the expected[3] contrast functions on signal-and-noise presentations and on noise-only presentations.

$$signal(x,y,t) = \langle c_{SN}(x,y,t)\rangle - \langle c_N(x,y,t)\rangle$$

The signal function is fixed, identical on every signal-and-noise presentation. For example, in Figure 2.1 the signal function of the 8 c/deg grating is

$$signal(x,y,t) = c\ e^{-(\frac{x}{4^\circ})^2-(\frac{y}{5.5^\circ})^2-(\frac{t}{70ms})^2} \sin(2\pi 8\frac{cycle}{degree}x).$$

In the continuous case (including my experiments) the contrast function of the signal-and-noise presentation was the sum of the contrast function of the signal and the contrast function of the noise:

$$c_{SN} = signal(x,y,t) + noise(x,y,t).$$

In the Poisson case the contrast function of the signal-and-noise presentation differs slightly from the sum of the contrast function of the signal and the contrast function of the noise:

$$c_{SN} \approx signal(x,y,t) + noise(x,y,t),$$

but we will shortly see that we may usually ignore the discrepancy. In terms of the contrast functions we have just defined, the original luminance functions are

[3] Expected means the average over the ensemble of possible luminance functions for that type of presentation (i.e. noise-only or signal-and-noise). This operation will be symbolized by angle brackets, < >.

$$L_N(x,y,t) = L_{mean} \times [1 + noise(x,y,t)]$$

and

$$L_{SN}(x,y,t) = L_{mean} \times [1 + signal(x,y,t) + noise(x,y,t)].$$

The constant of proportionality in the relation

$$c^2 \propto n^2_{critical} + n^2$$

is a measure of efficiency of detection. There is a body of theorems called Theory of Signal Detectability which establish the ideal performance attainable in various tasks involving detection of a signal in additive, Gaussian, white noise (Peterson, Birdsall, and Fox 1954). These theorems show that the detectability of a known signal by an ideal observer depends only on a dimensionless quantity called the signal-to-noise ratio, s/n, where n is the positive square root of n^2, the spectral density. I will shortly define the "signal energy" s^2.[4] The original definitions are all for functions of only one dimension, time. We will want to consider ideal detection of signals in noise of several dimensions, particularly dynamic, two-dimensional, white noise.

The contrast, c, of a contrast function is half its peak-to-peak amplitude.[5] The power, c^2_{rms}, of a contrast function is its variance,

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[4] The symbols s^2, n^2, and s/n are my notation. The conventional symbols, E, $N_0/2$, and $\surd(2E/N_0)$, were developed for functions of one dimension (time) and become very clumsy if generalized to functions of k dimensions: E, $N_0/2^k$, and $\surd(2^k E/N_0)$. See Appendix 1.

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[5] This follows from the $(L_{max}-L_{min})/L_{mean}$ definition of contrast, and the definition of the contrast function.

i.e. average squared value. The <u>signal energy</u>, s^2, is the integral of the squared signal function along the dimensions of random variation of the noise.[6] For example, if the noise is two-dimensional, dynamic, white noise, the signal energy is integrated over all space (i.e. the visual field) and time:

$$n^2(f_x,f_y,f_t) = n^2$$

$$s^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \text{signal}^2(x,y,t)\ dx\ dy\ dt.$$

If the noise is two-dimensional, static, white noise, the signal energy is integrated over the visual field:

$$n^2(f_x,f_y,f_t) = n^2(f_x,f_y)\ \delta(f_t)$$

$$n^2(f_x,f_y) = n^2$$

$$s^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \text{signal}^2(x,y)\ dx\ dy.$$

---

[6] This definition is clumsy because I wish to include noise which is white over various numbers of dimensions. When the noise is two-dimensional dynamic white noise my definition reduces to that used by Watson, Robson, and Barlow (1981) who define "contrast energy" of a contrast function as the product of its power and area and duration, i.e. the integral of the squared contrast function over space and time. Similarly, for static two-dimensional white noise my definition reduces to Linfoot's (1964) integral of the squared contrast function over space. It is the dimensionless signal-to-noise ratio that is the fundamental quantity. A more general treatment would allow any noise spectrum, $n^2(f_x,f_y,f_t)$ and signal spectrum $S^2(f_x,f_y,f_t)$ and define the squared signal-to-noise ratio as

$$(s/n)^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{S^2(f_x,f_y,f_t)}{n^2(f_x,f_y,f_t)}\ df_x\ df_y\ df_t.$$

This definition shows that the signal-to-noise ratio will be infinite if the signal has energy at any frequency at which there is no noise. The disadvantage of the general approach, for our purposes, is that the concept of signal energy is lost entirely. The compromise taken in this chapter has been to restrict the noise to be white in the dimensions in which the noise has random variation. Because of that restriction, the definition of signal energy adopted in this chapter will give us the same signal-to-noise ratio as the more general definition.

For many purposes it will be sufficient to use the fact that signal energy is proportional to squared contrast,

$$s^2 \propto c^2,$$

without going to the trouble to calculate the signal energy.

The Theory of Signal Detectability established the ideal detectability of signals in additive Gaussian white noise. Rose (1942, 1948) and de Vries (1943) developed fluctuation theory to handle the Poisson case[7]. The signal-to-noise ratio in the Theory of Signal Detectability is a great convenience: in the context of a detection experiment the stimuli may be characterized by the signal-to-noise ratio, and the observer's performance at many tasks may be characterized by the signal-to-noise ratio, d', that an ideal observer would require to perform equally well.[8] Corresponding theorems have not been developed for the Poisson case, and in general treatments of the Poisson case there is nothing even analogous to a signal-to-noise ratio. For example Barlow (1977) defines "quantum efficiency" as "the minimum proportion of quanta entering the eye that would enable the task to be performed." However, because it is so useful to have a single parameter to characterize

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[7] Fluctuation theory corresponds more closely to Signal Detection Theory (SDT, e.g. Green and Swets 1966) than to Theory of Signal Detectability (TSD, e.g. Peterson, Birdsall, and Fox 1954). Fluctuation theory and SDT both draw on proofs about ideal detectability to make models of human performance, often modified to take account of our imperfections. Thus while the TSD and Poisson-case theorems (e.g. Thibos, Levick, and Cohn 1979) about ideal detectability are unlikely to be wrong, fluctuation theory and SDT are speculative models of human behavior, and like most models, probably are wrong.

---

[8] This is Tanner and Birdsall's (1958) definition of d', not Green and Swets's (1966), but the two are numerically equal for the purposes of this chapter.

ideal detectability, fluctuation theorists usually make some approximations which allow them to calculate a signal-to-noise ratio.

Consider ideal detection of a bright disc of contrast c, area A, and duration T, on a dot display with average dot rate J (e.g. dots per deg^2 sec). The analysis would be identical for a display with no luminance noise, exchanging the word "photon" for "dot". The ideal detector would count the number of dots arriving in the area and duration corresponding to the signal. The count on a noise-only presentation has mean and variance ATJ; the count on a signal-and-noise presentation has mean and variance ATJ+2cATJ.[9] An exact calculation would determine the performance attainable from decisions based on these Poisson random variables. However, if ATJ is large the counts will, by the Central Limit theorem, have nearly Gaussian distributions, and if the contrast is low the variances of the two counts (N and SN) will be negligibly different (Cohn, Green, and Tanner 1975). With these approximations the performance of the ideal depends only on the mean difference 2cATJ in units of the common standard deviation $\sqrt{(ATJ)}$,

$$\frac{\mu}{\sigma} = \frac{2cATJ}{\sqrt{(ATJ)}} = 2c\sqrt{(ATJ)}.$$

Thus fluctuation theorists determine a "signal-to-noise ratio": the mean number of extra photons in the signal divided by the standard deviation of the noise-only count.

This is identical to the signal-to-noise ratio s/n that we would have calculated if we had applied our definitions of signal energy, s^2, and noise

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[9] The factor of 2 that appears here and throughout the rest of the calculation results from the definition of contrast as $(L_{max}-L_{min})/L_{mean}$, so that

$$\Delta L/L_{mean} = 2c.$$

spectral density, n^2, in the first place. The signal energy is $s^2=(2c)^2AT$, the noise spectral density is $n^2=1/J$, and so the signal-to-noise ratio is

$$\frac{s}{n} = \frac{2c\sqrt{(AT)}}{\sqrt{(1/J)}} = 2c\sqrt{(ATJ)}.$$

This shows that s/n is an important quantity, but we can do better. By considering the limits of the observer (that is, the bandwidth of the mechanism under study) we can show that the basic assumptions of Theory of Signal Detectability are usually satisfied to a good approximation by both continuous and dot displays. Then we can conclude that the signal's ideal detectability depends only on the signal-to-noise ratio, s/n. We must establish the validity of applying theorems which assume additive, Gaussian, white noise to experiments in which the noise was non-Gaussian, and in some cases(the dot experiments) non-additive.

The discrepancy between the noise assumed in theory and used in practice results in part from physical constraints on luminance displays. For example, a Gaussian amplitude distribution would require an infinite dynamic range, but excursions from the mean luminance on any actual display have some positive limit, and negative excursions cannot go below zero luminance. However we can make use of the fact that the observer is only sensitive over a finite bandwidth of spatial and temporal frequencies to show that the effect of the experimentally produced noise is the same as that of the noise assumed in the theory. In the same way that a small aperture limits the resolution of a camera, the finite bandwidth of a mechanism under study limits its resolving power. If the sample density (or dot flux) is many times the bandwidth of the mechanism under study then the mechanism cannot resolve individual samples (or dots) and responds only to weighted sums over many samples. The amplitude distribution of these sums, in accord with the Central Limit Theorem, approaches Gaussian form. If the sample density is many times the bandwidth of the mechanism under study then the effects of the noise depend only on its spectrum $n^2(f)$ over the frequencies of interest. Thus the validity of the continuous Gaussian noise assumption rests on establishing a low upper bound estimate of the bandwidth of the mechanism under study. We could conservatively estimate this bandwidth as the bandwidth of the observer, but there is evidence that the performance of an observer detecting a given signal is affected only by noise within a much narrower bandwidth.

Let us take Robson's (1966) contrast sensitivity function over spatio-temporal frequency to estimate the observer's (two-sided) bandwidth at about 2×30 c/deg × 2×30 c/deg × 2×30 Hz ≈ 200,000 per deg^2sec. On this basis we can expect that the observer will be unable to distinguish dot fluxes large relative to 200,000 per deg^2sec from continuous noise with the same spectral density. Alternatively we could use the evidence discussed in Chapter 3 that the threshold of an observer detecting a grating is affected only by noise in the vicinity of the grating frequency, to estimate the bandwidth of a hypothetical, tuned mechanism, a "channel", which mediates detection. Though the experimental techniques were varied, the derived bandwidths are suitable for at least a rough calculation. These bandwidths are ±1 octave about the spatial frequency of a signal grating, .6 c/deg orthogonally, and about 16 Hz temporally. For a 4 c/deg grating, the product would be 2×6 c/deg × 2×.6 c/deg × 2×16 Hz ≈ 500 per deg^2sec. Thus on the basis of the noise-masking results of Chapter 3 we would expect the mechanism that mediates detection of 4 c/deg to be equally affected by dot fluxes large relative to 500 per deg^2sec and continuous noise with the same spectral density. For a static dot display similar calculations yield a (two-sided) bandwidth of about 4,000 per deg^2 for the observer, and 14 per deg^2 for a 4 c/deg channel. The Results section will show that thresholds in continuous and dot noise are very similar for dot fluxes down to several hundred dynamic dots per deg^2sec, and about 18 static dots per deg^2 suggesting that the channel bandwidth, not the observer's bandwidth is what matters. Below these dot fluxes and densities observers are much more accurate, and it has been suggested that they base their decisions on numerical estimates of dot number.

One of the results of explicit consideration of the finite-bandwidth of the mechanism under study is that contrast is an inappropriate measure of wide-band noise. It was mentioned above that a finite bandwidth mechanism cannot distinguish between the noises actually used, which had finite contrast, and noise with a Gaussian amplitude distribution and infinite bandwidth, whose contrast would be infinite. Stimuli whose bandwidth are narrow relative to the mechanism under study may be usefully characterized by their power, c_{rms}^2, or rms contrast c_{rms}; stimuli whose bandwidths are broad relative to the mechanism under study are characterized by their spectral density $n^2(f)$.

In the Poisson case the observer must detect the relative increase or decrease in events (dots or photons) in some areas due to the "signal". The likelihood of an event in a given part of the field is determined by a prior probability. In noise-only presentations the events can appear anywhere over the field with equal prior probability. In the signal-and-noise presentations the prior probability is modulated by the signal function, signal(x,y,t).

The contrast functions of the signal and noise are not additive in the Poisson case. The spectral density of the noise-only presentation is given by 1/J, the reciprocal of the event flux, but in the signal-and-noise presentation the event flux is modulated by the signal and therefore the spectral density of the noise is modulated too. In principle this could be relevant to detection, i.e. distinguishing signal-and-noise from noise-only presentations. Since most of the signal contrasts (and noise modulation) were below 10%[10], the modulation of noise spectral density will be neglected, so that we may assume additivity of the signal and noise.

Thus we have seen, firstly, that the theoretical assumption that the noise is white requires that noise in our experiments have a constant spectral density over the bandwidth of the mechanism under study, which may be much less than the observer's bandwidth. Secondly, the theoretical assumption that the noise is Gaussian requires that we use a sample flux much greater than the bandwidth of the mechanism under study. Finally, the theoretical assumption of additivity of signal and noise is not quite satisfied in the Poisson case, but at low signal contrasts the discrepancy is probably negligible. As we will see in the Review, these expectations are confirmed by the essentially identical results with continuous and dot experiments.

Now we can introduce <u>d'</u>, the signal-to-noise ratio, s/n, that would be required by an ideal observer to perform as well as the observer under study (Tanner and Birdsall 1958)[11]. d' may be calculated, for example, from the proportion correct of 2IFC responses, or from the hit and false-alarm rates of yes-no responses.

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[10] Apparently no one has measured how well we detect modulation of rms noise contrast. Dudley (personal communication) of the Royal Aircraft Establishment, Farnborough, England, has suggested it may be an important cue in detecting targets on image intensifiers (which are dot displays). But this seems unlikely. Extrapolating from detection of contrast increments of sinusoidal gratings would suggest a Weber fraction (i.e. threshold increment as a fraction of the background) no less than 30%. Since the spectral density of the noise is proportional to contrast squared, this would correspond to a $(1.3/1)^2 \approx 1.5$-fold increase, or a 50% Weber fraction of spectral density. This suggests that the 10% modulation caused by a signal at 10% contrast is utterly undetectable by the observer.

Efficiency

Conceptually, the "efficiency" of a detector is the squared signal-to-noise ratio of its output, $(s/n)^2_{out}$, divided by the squared signal-to-noise ratio at its input, $(s/n)^2_{in}$,

$$\frac{(s/n)^2_{out}}{(s/n)^2_{in}}.$$

It is important to realize that the reward for our labor in establishing the applicability of Theory of Signal Detectability is the knowledge that no transformation can increase a signal-to-noise ratio. Efficiency cannot exceed 1.

The output is often not of the same nature as the input (e.g. the observer's responses have little resemblance to the pattern he may be detecting) so that it is not obvious how to calculate the output signal-to-noise ratio. Efficiency is defined as the ratio of the squared signal-to-noise ratio which would have been required by an ideal device for the observed performance, divided by the squared signal-to-noise ratio at input (Tanner and Birdsall 1958),

$$\text{Efficiency} = \frac{(s/n)^2_{ideal}}{(s/n)^2_{in}}.$$

---

[11] This is not the same as Green and Swets's (1966) definition which treats the ideal observer (of a signal known exactly) as a model of the human observer and then defines d' as a parameter of the internal statistics of that model. Green and Swets's definition would be unsatisfactory for our current purposes because it would make our conclusions appear to depend on the validity of an unlikely model. The two definitions of d' are numerically equal for exactly known signal, so the calculations in this chapter would be the same for either definition, but differences will appear in Chapter 5, which considers detection of signals not known exactly.

We have already defined d' as the signal-to-noise ratio the ideal would have required:

$$d' \equiv (s/n)_{ideal},$$

so we can write the definition of efficiency more concisely as

$$\text{Efficiency} = \frac{(d')^2}{(s/n)^2},$$

where the subscript "in" has been dropped because it is implied anyway.

Since the efficiency definition depends only on a ratio including signal energies and noise levels we may equate either the signal energies or noise levels without constraining the ratio that defines efficiency. If we equate the noise levels the efficiency is the ratio of signal energies:

$$\text{Efficiency} = \frac{s^2_{ideal}/n^2}{s^2/n^2} = \frac{s^2_{ideal}}{s^2}.$$

In fact this is Tanner and Birdsall's (1958) original definition of ("detective") efficiency, the fraction of the energy made available which is ideally required.

Similarly we may equate the signal energies and write efficiency as a ratio of noise levels:

$$\text{Efficiency} = \frac{s^2/n^2_{ideal}}{s^2/n^2} = \frac{n^2}{n^2_{ideal}}.$$

If the input is made up of discrete stochastically independent (Poisson) events then we may equate the signal energies and use the equality of spectral density and reciprocal event rate,

$$n^2 = 1/J,$$

to write efficiency as a ratio of event rates:

$$\text{Efficiency} = \frac{n^2}{n^2_{ideal}} = \frac{J_{ideal}}{J}.$$

Calculation efficiency

Let us call the sum of actual and equivalent noise levels the effective noise level. Since in practice we estimate the equivalent noise by $n^2_{critical}$ we have

$$n^2_{effective} = n^2 + n^2_{critical},$$

so our relation,

$$c^2 \propto n^2_{critical} + n^2,$$

can be more succinctly stated as proportionality of squared threshold to effective noise level,

$$c^2 \propto n^2_{effective}.$$

Define the effective signal-to-noise ratio as the square root of the ratio of signal energy to effective noise spectral density,

$$(s/n)^2_{effective} = \frac{s^2}{n^2_{effective}} = \frac{s^2}{n^2_{critical} + n^2}.$$

Define the Calculation Efficiency, CE, as the ratio of the squared signal-to-noise ratio which would have been required by an ideal device for the observed performance, divided by the squared effective signal-to-noise ratio at input,

$$CE = \frac{(d')^2}{(s/n)^2_{effective}} = \frac{(d')^2}{s^2/(n^2_{critical} + n^2)}.$$

Define the effective stimulus as the sum of (the contrast function of) the observer's equivalent noise and (the contrast function of) the actual stimulus. The calculation efficiency measures the observer's performance relative to how well an ideal observer would perform in response to the effective stimuli.

Returning to our empirical relation,

$$c^2 \propto n^2_{effective},$$

we may replace the squared contrast c^2 by the signal energy s^2, since they are proportional to each other, to get

$$s^2 \propto n^2_{effective},$$

so their ratio, $(s/n)^2_{effective}$, is constant. Furthermore, threshold in each data set that we will consider either corresponds to a constant d', or, in the case of method-of-adjustment thresholds, we might suppose it does. So we may write

$$\text{constant} = \frac{(d')^2}{(s/n)^2_{effective}},$$

but this is the calculation efficiency, CE. So our relation is just the finding that the calculation efficiency is constant:

$$\text{constant} = CE.$$

In general the constant will depend on d'. Thus, the empirical finding is that for a given d' the calculation efficiency is independent of the noise level, n^2. At high noise levels, $n^2 \gg n^2_{critical}$, the detective and calculation efficiencies will be asymptotically equal.

So far we have observed that the data in Figure 2.1 were described well by the relation

$$c^2 \propto n^2_{critical} + n^2.$$

The definition of calculation efficiency may be rearranged to write

$$s^2 = \frac{(d')^2}{CE}(n^2_{critical} + n^2).$$

This redeems the promise to explain the additive constant (it is $n^2_{critical}$) and the proportionality constant (explained by CE) of the linear relation found between squared contrast and noise level.

Transduction and quantum efficiencies

Having referred the observer's noise to his visual field we may now ask what is responsible for it. Is it due to photon noise?

Let $J_{photons}$ represent the retinal illuminance in corneal[12] photons per sec deg^2.[13] It is shown in Appendix 1 that a distribution made up of many independent (Poisson) events has a white spatio-temporal spectrum and a spectral density equal to the reciprocal of the event rate. The noise results from the probabilistic way that light is absorbed and emitted by matter, in this case our photoreceptors, so that a uniform retinal illuminance results in a noisy photon flux. We don't know what fraction of the photons is absorbed - to some extent that is the purpose of the enterprise - but we can calculate a minimum noise level corresponding to complete utilization. Define the photon noise, $n^2_{photons}$, as that minimum noise level:

$$n^2_{photons} = 1/J_{photons}$$

An otherwise noise-free device that used all the photons would have an equivalent input noise,

$$n^2_{critical}=n^2_{photons}=1/J_{photons};$$

if it used half the photons its equivalent noise would be twice that,

$$n^2_{critical}=2n^2_{photons}=1/(\tfrac{1}{2}J_{photons}).$$

Thus it seems reasonable to call the ratio of the photon noise $n^2_{photons}$ to the observer's equivalent noise, his "transduction efficiency", "TE". The observer may in fact absorb more photons but he must absorb at least the fraction TE of the photons for his equivalent noise to be as low as it is. By the end of this

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[12] Corneal photons are those photons which arrive at the observer's cornea(s) and will not be blocked by his iris(es). We will follow Barlow (1962a) and count all corneal photons, whether presented monocularly, or distributed binocularly to both the observer's eyes. This will make it necessary to halve quantum efficiency estimates such as those of Rose (1948) or Jones (1959) which neglected the light gathered by the observer's second eye.

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[13] If the light is not monochromatic it is conventional to base quantum efficiency calculations on the photon flux at the wavelength of maximum sensitivity (i.e. 507 nm for scotopic vision, 555 nm for photopic vision) which would produce the same photopic or scotopic retinal illuminance. Scotopically we are monochromats, and the equivalence follows. Photopically, van Nes and Bouman (1967) have shown that threshold contrasts of sinusoidal gratings (.5 to 50 c/deg) were the same (after correcting for the dependence of diffraction on wavelength) in red, green, and blue monochromatic lights equated for photopic luminance by flicker photometry.

section it should be clear to the reader that transduction efficiency is what Rose (1946, 1948) called the "quantum efficiency of the eye". Rose (1948) claimed our transduction efficiency was between .5% and 5% (for white light) for a wide range of conditions. Surprisingly there is still little evidence on this point, but we will see that available evidence yields estimates of the transduction efficiency that all lie between .4% and 3% (at 555 nm) for a wide range of patterns and luminances, in remarkably good agreement with Rose's claim.

The notions of transduction and quantum efficiency have an interesting history which will shortly be recounted. However it will be best to begin with a theoretical framework.

<u>Quantum</u> <u>efficiency</u>, <u>QE</u>, is efficiency of performance with respect to input in the case where the input noise is just photon noise,

$$QE = \frac{(d')^2}{(s/n)^2_{photons}}.$$

As in the general case we may fix the signal energy and rewrite the quantum efficiency as

$$QE = \frac{J_{ideal}}{J_{photons}},$$

which is "the minimum proportion of quanta entering the eye that would enable the task to be performed" (Barlow 1977).

Quantum efficiency (also called "detective quantum efficiency"[14]), is now the principal index of performance of light transducers in many fields. Apparently Rose (1946) invented it. This will be discussed further in the

History sub-section.

Calculation efficiency, CE, is efficiency of performance with respect to the effective signal-to-noise ratio:

$$CE = \frac{(d')^2}{(s/n)^2_{effective}}.$$

Transduction Efficiency, TE, is the ratio of the photon noise spectral density, $n^2_{photons}$, to the observer's equivalent noise spectral density measured in practice as $n^2_{critical}$,

$$TE = n^2_{photons}/n^2_{critical}.$$

If the input noise is just photon noise, then the effective noise level $n^2_{effective}$ equals $n^2_{critical}$ and we may write

$$TE = \frac{n^2_{photons}}{n^2_{effective}} = \frac{(s/n)^2_{effective}}{(s/n)^2_{photons}}.$$

These definitions imply that the quantum efficiency is the product of the transduction and calculation efficiencies,

$$QE = TE \times CE,$$

$$\frac{(d')^2}{(s/n)^2_{photons}} = \frac{(s/n)^2_{effective}}{(s/n)^2_{photons}} \times \frac{(d')^2}{(s/n)^2_{effective}}.$$

---

[1] Jones (1959) called it "detective quantum efficiency, DQE," to distinguish it from "responsive quantum efficiency" (ratio of events out to events in) which was in use then but not now. It seems safe now to drop "detective" and return to Rose's (1946) original term, "quantum efficiency".

No one has ever noted this fact before, even though all three quantities are in current use.

Figure 2.2 illustrates these relations and shows how they correspond to the traditional terminology which we will shortly see in its historical context. Traditionally the observer is represented by eye and brain. In the new terminology the traditional eye and brain are replaced by the two boxes labeled "transduction" and "calculation". The experimental stimuli themselves are the input to "transduction". The output of "transduction" is the input to "calculation", and the output of "calculation" is the observer's performance. Signal-to-noise ratios can be assigned to three points: stimulus, $(s/n)_{photons}$; output of transduction, $(s/n)_{effective}$ ("k" for Rose, 1948); and performance, d'. The various efficiencies differ only in which points they take as input and output. Transduction efficiency (Rose's "quantum efficiency of the eye") is the efficiency of the transduction box. Calculation efficiency (Barlow's "central efficiency") is the efficiency of the calculation box. Quantum efficiency is the efficiency of the two boxes taken together. In the literature on this topic the reader should be wary of the term "quantum efficiency of vision" which has been used inconsistently, and may mean either transduction efficiency or quantum efficiency. From here on the traditional terminology will bear quote marks, and the new terminology will not.

The only difference between the traditional model and the new model is in naming; the quantities are identical. The new names are more abstract because the anatomical division implied by the traditional names is unjustified, and further, because this analysis may be applied to all light-sensitive devices, not just human observers.

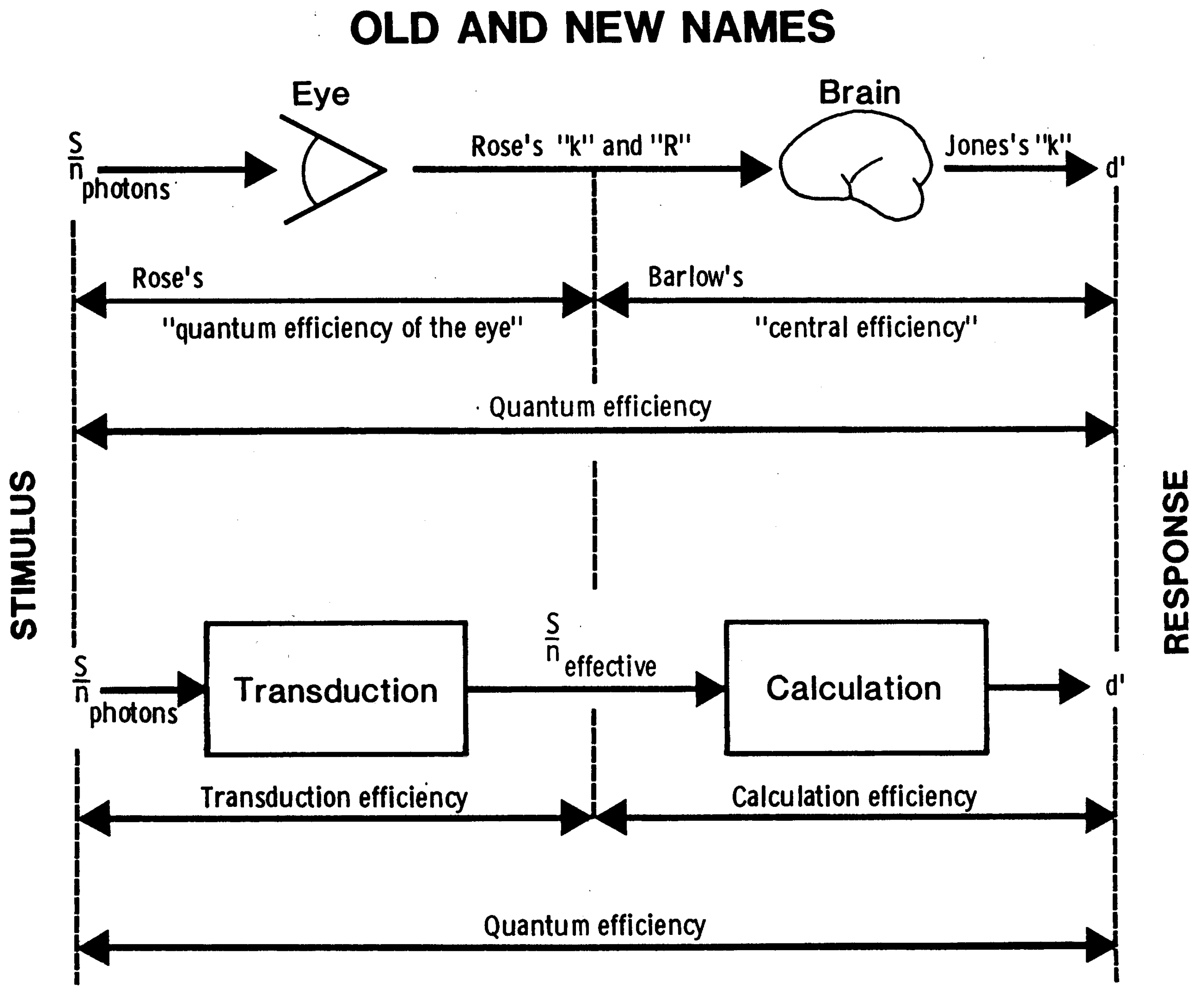
OLD AND NEW NAMES
Eye
Brain
S/n photons
Rose's "k" and "R"
Jones's "k"
d'
Rose's
"quantum efficiency of the eye"
Barlow's
"central efficiency"
Quantum efficiency
STIMULUS
RESPONSE
S/n photons
Transduction
S/n effective
Calculation
d'
Transduction efficiency
Calculation efficiency
Quantum efficiency

Figure 2.2

The names "transduction" and "calculation" are somewhat whimsical, and correspond only loosely to their literal interpretations: the physical business of converting radiation to some other form of energy, and the observer's conscious machinations to decide on a response.[15] There is some question as to whether the name "transduction efficiency" is appropriate, given its loose relation to the physical conversion of light. It describes the relation between the actual stimulus, which is made of light, and the effective stimulus which is a contrast function and has no associated photon noise. I am not aware of any other appropriate meanings for the term. The phrase "fraction of photons effectively absorbed" has been bandied about, but is hopelessly ambiguous when applied to anything as complex as a visual system, as Barlow (1958b) has pointed out:

> "[In the case of the eye, quantum efficiency is] the fraction of quanta sent through the pupil which are 'effectively absorbed' by the photosensitive materials subserving the mechanism under consideration, but a difficulty arises in deciding what to understand by the words 'effectively absorbed'. In the case of rhodopsin a quantum can be absorbed without bleaching the molecule, and even if bleaching occurs it is by no means certain that the rod is always activated. Further, if the rod is activated it is still not certain that this information is succesfully transmitted to the place where the threshold decision is made."

Barlow then went on to give the modern definition of quantum efficiency which we have already discussed. At the end he added,

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[15] For example consider detection of a large disc. The ideal detector counts photons over the area and duration of the possible signal. Now suppose we modify this detector by the incorporation at the image plane of either a neutral density filter with a transmission F, or a coarse grid (like mosquito screen) with the same transmission. Either will reduce the quantum efficiency by the factor F. However the neutral density filter will have reduced the transduction efficiency by F, without affecting the calculation efficiency, while the grid will not affect the transduction efficiency and will reduce the calculation efficiency by F.

> "This quantity [quantum efficiency] will, of course, always be smaller than the fraction of quanta actually absorbed, but it might approximate to it in the case of a task to which the eye is well adapted. One might hope to subdivide the overall quantum efficiency into the efficiencies of the various steps - absorption, excitation of receptors, transmission to the central nervous system, etc., - but this is clearly not possible until information becomes available about the stages intermediate between the light entering the eye and the subject giving his response."

Transduction and calculation efficiencies do offer such a subdivision, though not along the anatomical lines Barlow hoped for.

When applying these measures to the human observer we have taken the visual field as the input. As a result the transduction and quantum efficiencies incorporate the losses due to the optics of the eye (i.e. absorption and blurring). Thus these are the observer's transduction and quantum efficiencies with respect to the observer's visual field. If we had an accurate description of the retinal image we could instead determine the transduction and quantum efficiencies with respect to the retinal image and thus exclude the optical losses.

Quantum, calculation, and transduction efficiencies may be determined for all light-sensitive devices, but only quantum efficiency has been applied outside of human vision. For human obsevers, Barlow (1962b) reported 5% quantum efficiency for contrast discrimination of an optimized brief disc presented 15° in the nasal field of the dark-adapted eye. Barlow, Levick, and Yoon (1971) reported quantum efficiencies of 5% to 17% for dark-adapted retinal ganglion cells in the cat. Fellgett (1958) and Jones (1958) reported quantum efficiencies of 1% for photographic film (Plus-X and Tri-X), and Jones (1959b) reported 3% for a tv camera (an Image Orthicon).

A brief history of these ideas will now be presented.

A brief history of transduction and quantum efficiencies.

1. Radio engineering and radiation detectors

Rose (1946) is generally credited with inventing quantum efficiency (Fellgett 1958, Jones 1959, Dainty and Shaw 1974), and we may further credit him with inventing transduction efficiency (which he called "quantum efficiency of the eye"). Bearing in mind that Rose worked at the RCA (Radio Corporation of America) Laboratories it is interesting that analogous quantities to transduction efficiency and quantum efficiency appeared in the radio engineering literature in the early 1940's.

Radio waves, like light, are electromagnetic radiation. Radio receivers and tv cameras (and the human eye for that matter) are radiation detectors. In analyzing their performance however, a difference appears. At radio wavelengths each photon carries little energy, and radio receivers are limited by thermal noise from the antenna (in the absence of other noise in the environment), while visible-light photons each carry much energy and visible-light detectors are usually limited by the quantal nature of light.

In his paper entitled, "The sensitivity performance of the human eye on an absolute scale", Rose (1948) acknowledged "many discussions of the subject of this paper with Dr.D.O.North of these [RCA] laboratories." Six years earlier North (1942) had published an article entitled, "The absolute sensitivity of radio receivers".

In that article North showed how to calculate the thermal noise of an antenna of specified temperature, and showed how to use a dummy antenna to measure the receiver's equivalent input noise with respect to the antenna. Let us represent these quantities as $n^2_{thermal}$ and $n^2_{equivalent}$. North then defined the "noise factor" as their ratio,

$$\text{noise factor} = n^2_{equivalent}/n^2_{thermal}.$$

Except for the substitution of thermal noise for photon noise this is the reciprocal of the transduction efficiency,

$$1/TE = n^2_{critical}/n^2_{photons},$$

where $n^2_{critical}$ is our empirical estimate of the observer's equivalent input noise.

Jones (1959b) pointed out that one might choose to define the ambient illumination as including black body radiation corresponding to the temperature of the sensor itself. This would unify consideration of all radiation detectors. The black-body radiation is negligible in human vision and tv cameras, but is very important in thermocouples and radio receivers.

The origin of quantum efficiency may lie in a later paper by Friis (1944),

> "A rigorous definition of the noise figure of radio receivers is given in this paper. The definition is not limited to high-gain receivers, but can be applied to four-terminal networks in general [i.e. with an input and an output]. . . .
>
> "The noise figure F of the network is defined as the ratio of the available signal-to-noise ratio at the signal-generator terminals [i.e. input] to the available signal-to-noise ratio at its output terminals." [The signal-to-noise ratios are not squared because they are already ratios of powers.]

In essence this is the reciprocal of quantum efficiency. Jones (1959b, pp. 100-101) has noted this parallel,

> " . . . the reciprocal of Q_D [quantum efficiency] is a kind of noise figure. $1/Q_D$ differs from the usual noise figure, however, in that the reference noise is radiation noise≡photon noise, whereas the reference noise of the ordinary noise figure is Johnson [i.e. thermal] noise."

"Noise figure" and "noise factor" are still used interchangeably in radio engineering, because radio receivers are comparatively simple devices, and, in our terminology, they generally have a calculation efficiency of 1. Thus Friis considered his "noise figure" a more rigorous version of North's "noise factor", though we may now see their two quantities as special cases of two measures which are in general different (because, in general, the calculation efficiency may be less than 1).

On largely empirical grounds, Jones (1947, 1949a,b) defined a series of "factors of merit" M_1, M_2, etc. for radiation detectors of which M_1 is related to the radio engineer's "noise figure" (indeed Jones considered the special case of a dipole antenna with his treatment of radiation detectors). However the discussion was limited to devices (like thermocouples and radio antennas) limited by their own thermal noise, and did not consider the stochastic nature of the radiation itself. Later Jones (1959b) dropped his "factors of merit" in favor of quantum efficiency, which he attributes to Rose (1946). Jones made the first determinations of the quantum efficiency of television cameras (Jones 1959b), and human vision (Jones 1959), and was simultaneous with Fellgett (1958) in determining the quantum efficiency of photographic film.

In connection with our analogy to transduction efficiency it is interesting to read Davenport and Root's (1958, p. 209) summary,

> "The noise figure of an amplifier is thus a measure of the noisiness of the amplifier <u>relative to the noisiness of the source</u>." [emphasis in original]

It could be argued that "transduction efficiency" is too specific and that a better and more general term would be "noise fraction", defined as the fraction of the equivalent input noise which is due to the source. This would be applicable to vision and to radio receivers (where it would equal the reciprocal of the noise figure). Without detracting from the importance of

Rose's invention of quantum and transduction efficiencies, we have seen that analogous ideas had been applied to radios by his colleagues.

A brief history of transduction and quantum efficiencies.

2. Vision

Let us begin with the Rose-de Vries law. The studies of detection of light in the dark by Hecht, Shlaer, and Pirenne (1942) and others have been well-reviewed by Barlow (1962a), and are not directly relevant here.

Rose (1942) and de Vries (1943) developed fluctuation theory to calculate the signal-to-noise ratio of Poisson events. Both assumed that some fraction of the corneal photons would be transduced by the eye and that threshold occurred when the signal-to-noise ratio based on transduced photons was 1. This was sufficient to predict the Rose-de Vries law, which is essentially a claim that the observer's quantum efficiency is constant for detection of spots of all diameters and all background luminances.

The Rose-de Vries law is now usually stated as

$$c\sqrt{(AIT)} = \text{constant},$$

where c is the threshold contrast of a disc, A is its area, T is its duration, and I is the background retinal illuminance. It states that the stimulus signal-to-noise ratio is constant at threshold,

$$(s/n)_{\text{photons}} = \text{constant}.$$

When the thresholds all have the same d' the Rose-deVries law implies that the quantum efficiency is constant,

$$QE = \frac{(d')^2}{(s/n)^2_{\text{photons}}} = \text{constant},$$

but we are jumping ahead, the concept of d' did not yet exist.

Rose (1948) later acknowledged that the Rose-de Vries law did not provide a satisfactory basis for determining the fraction of corneal photons that are transduced because of the arbitrary nature of the assumption that threshold always occurs at an effective signal-to-noise ratio of 1. To remedy this flaw he devised a brilliant indirect way of measuring the effective signal-to-noise ratio at threshold. According to Rose the eye transduces photons to nerve impulses, and the brain counts nerve impulses. This is schematically represented in Figure 2.2. Rose (1948b) said,

> "To name television pickup tubes, photographic film, and the human eye is to name examples of the three major types of visual processes; electrical, chemical, and biological. The detailed mechanics of the operation of a television pickup tube has little in common with that of photographic film and even less, perhaps, in common with that of the human eye. This, in spite of literary references to television pickup tubes as 'electric eyes' and in spite of pictorial explanations of the human eye confined to the parallelism of parts - lens, black box, and film - of an ordinary camera. What is common to all of these devices, especially if they are well designed, is some means for counting incoming light quanta. The particular means for, or mechanics of, counting may be as varied as the imagination of man and nature combined. Television tubes generally convert light quanta into photoelectrons and count the number of electrons by measuring a current or a voltage. Photographic film, by an intermediate chemical process, converts each quantum (or at most a small number of quanta) into an opaque granule of silver and observes the density of the silver deposit as a measure of the number of granules. The human eye, also, by an obscure intermediate process, converts a small number of incoming quanta into a single nerve pulse. The brain estimates the number of original quanta by the frequency of arrival of these nerve pulses and (a speculative thought) by the regularity of their arrival."

Rose then proceeded to determine the "quantum efficiency of the eye".

Rose (1946, 1948, 1948b) never explicitly defined quantum efficiency, only saying it was the "quantum yield of the photo process" (Rose 1946). However for tv cameras he calculated the quantum efficiency by the fraction of the photons that an "ideal photo pickup device" would have required to produce the observed performance, which is consistent with our definition. Thus it is reasonably clear that when Rose sought to determine the "quantum efficiency of the eye" he meant quantum efficiency in the same way we do,[16] but he never explained what he meant by the "eye" and "performance of the eye". He knew how to determine the signal-to-noise ratio at the input, he had already done that for tv cameras and photographic film, but there is no obvious way to measure the signal-to-noise ratio at the eye's output. The later approach taken by Jones (1959) and Barlow (1962a, b) was to measure the observer's reliability in detection and use the Theory of Signal Detectability (Peterson, Birdsall, and Fox, 1954) to assign a signal-to-noise ratio, d', to the measured performance. That tells us the quantum efficiency of the whole observer[17], but Rose wanted to know the quantum efficiency of just the eye, for comparison with other picture-pickup devices.

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[16] In fact Rose (1946) is generally credited with inventing quantum efficiency (Fellgett 1958, Jones 1959, Dainty and Shaw 1974, p. 28).

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[17] Barlow (1958b) seems to have been the first to think of measuring the quantum efficiency of an observer. While Rose (1946, 1948, 1948b) did measure the quantum efficiency of tv cameras and suggested its application to photographic film he never (in the published papers I am aware of) suggested calculating a signal-to-noise ratio from the observer's detection performance, i.e. d'. That calculation, based on Theory of Signal Detectability, is foreign to Rose's approach; he talked about image transduction properties of the eye, not detection performance of the observer. The question is who first thought of using Theory of Signal Detectability (TSD) to apply quantum efficiency to human observers. Hecht, Shlaer, and Pirenne (1942) are excluded because their analysis ignored false positives, an inconceivable omition in the context of TSD. Tanner and Birdsall's (1958) definition of the detective efficiency as a ratio of energies (which a ratio of photons certainly is) is literally the modern definition of quantum efficiency, though they were speaking in the context of audition. Barlow (1958b) defined,

> "the <u>overall</u> <u>quantum</u> <u>efficiency</u> as the smallest fraction of the quanta sent into the eye which would enable it to perform the task which it does perform."

Jones (1959) was the first to publish estimates of human quantum efficiency, but he cited a 1957 letter from Barlow on the topic.

Rose measured the signal-to-noise ratio of the "eye's performance" by an ingenious indirect method. Rose (1946) said,

> "The smallest observable signal-to-noise ratio has often been taken to be unity [e.g. by Rose 1942, and de Vries 1943]. Actually, by virtue of its statistical origin, the smallest observable R is a function of how often one prefers to have his observations correct. This much is verifiable both from analysis and from the use of physical instruments as observers. . . .
>
> "Human Eye. - [Quantum efficiency] is not immediately applicable to the human eye because there is no way of directly measuring the signal-to-noise ratio that the brain perceives. . . . This defines the constant to be equal to the minimum perceivable value of R. . . . The determination of this constant is of considerable importance in estimating the quantum efficiency of the human eye and deserves more experimental work."

As Figure 2.2 shows, what Rose calls "R", "the signal-to-noise ratio that the brain perceives" is the effective signal-to-noise ratio.

We have argued that in the presence of a high level of luminance noise the luminance noise would dominate other noises and the effective signal-to-noise ratio equals the signal-to-noise ratio at the display. Rose (1948) gave a reciprocal but equivalent argument. In describing his kinescope he said,

> "The gain in the multiplier section is sufficient to make each photo electron, liberated from the photo-cathode, visible on the kinescope screen as a discrete speck of light. That is, each quantum usefully absorbed at the primary photo surface can be counted in the final picture."

Referring to a dot display with discs of various sizes and contrasts, he said,

> "The threshold signal-to-noise ratio has this meaning. Take the smallest black (not grey) disk that may be seen in any one of the pictures. Transpose the outline of the disk to the neighboring white background. Count the average number of 'quanta' (specks of light) within this outline. The average number of 'quanta' is the signal associated with the black disk; the square root of the average number is the root mean square fluctuation, and the ratio of signal to r.m.s. fluctuation, also the square root of the average number, is the threshold signal-to-noise ratio. A similar operation can be carried out for any of the grey dots to obtain the same value of k. The result of this operation are that k lies in the neighborhood of 5."

He found the signal-to-noise ratio at threshold to be about 5 for discs of various sizes in static and dynamic dot images and settled on that value.

Rose's description of the operation of the "eye" and his method of determining the signal-to-noise ratio of its performance show that Rose's term, "eye", designates the transduction process, not the organ. However to properly determine the "quantum efficiency of the eye" Rose should have measured the effective signal-to-noise ratio at threshold for each of Blackwell's disc sizes at each luminance. Instead, Rose made a few informal measurements and then concluded that the effective signal-to-noise ratio at threshold is always about 5. This allowed him to calculate the "quantum efficiency of the eye" from Blackwell's (1946) thresholds for discs (assuming the eye's "storage time" is .2 seconds and estimating the pupil sizes from Reeves's, 1920, tables). With these assumptions Rose found that all of Blackwell's threshold contrasts (discs 2' to 100' in diameter over a luminance range of 3×10^{-6} to 3×10^{2} cd/m^2) led to quantum efficiency estimates (for white light) bounded by 5% at low luminances and .5% at high luminances[18].

Rose acknowledged that there were deviations. The "quantum efficiency of the eye" at each luminance was lower for the smaller and larger discs. However he remarked that the variation of "quantum efficiency of the eye" was only tenfold over an enormous range of light levels and disc diameters. He explained that this ruled out reduction in quantum efficiency (by bleaching of visual pigment) as a primary means of light adaptation. Thus Rose produced a

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[18] For direct comparison with the transduction efficiency estimates of Figure 2.9 Rose's estimates should be halved, because he ignored the light received by the second eye, and approximately tripled, because he did not weight the photons by the photopic luminosity function, as we will. Given the approximate nature of the rest of his assumptions a factor of 1.5 does not seem worth bothering about.

dramatic result: the "quantum efficiency of the eye" was between .5% and 5% over a vast range of conditions. However if he had carefully measured the threshold contrast signal-to-noise ratio of discs on his dot display he would have found, as Sturm and Morgan (1949) later did, that threshold varied with disc size in much the same way as Blackwell's data, smaller and larger discs having higher threshold signal-to-noise ratios. This contradicts Rose's assumption that the effective signal-to-noise ratio is constant, and, had Rose taken it into account he would have had to reduce the tenfold range over which he said the transduction efficiency varies.

Thus, conceptually, Rose's "quantum efficiency of the eye" is the transduction efficiency, but in practice Rose blurred the distinction between it and quantum efficiency by assuming that the calculation efficiency is constant; that is, that the transduction efficiency is a multiple of the quantum efficiency.[19] Neither Rose nor anyone since seems to have realized that "quantum efficiency of the eye" and quantum efficiency of the observer might depend very differently on signal parameters and background luminance.

Aguilar and Stiles (1954) created conditions designed to favor rod vision over cone vision, and measured threshold for a large (9°), brief (.2 sec) disc over a wide range of luminances. They found that the threshold contrast was constant from below .01 to above 100 scotopic trolands, whereas the Rose-de Vries law predicted that the 10,000-fold increase in luminance should have reduced the threshold contrast 100-fold. This implies a 10,000-fold fall in

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[19] Of course this can only be seen with hindsight. One can hardly expect Rose to have taken pains to distinguish the "quantum efficiency of the eye" from the quantum efficiency of the observer that Jones (1959) would introduce more than ten years later. It is possible that Rose was misled by the equivalence of noise figure and noise factor of radios to miss the fact that human vision has several kinds of efficiency.

quantum efficiency. It does not contradict Rose (1948) only because he restricted his claim of a nearly constant "quantum efficiency of the eye" to the range of parameters covered by Blackwell's data (maximum disc diameter was 100').

Barlow's (1958) classic demonstrations that the Rose-de Vries "law" is violated nearly everywhere showed that the quantum efficiency of the observer depends on signal contrast, size, duration, and background luminance, but these violations are not inconsistent with a constant transduction efficiency. This is because, contrary to Rose's assumption, the observer's calculation efficiency depends on the signal parameters (i.e. k changes), and may account for all the variation of quantum efficiency in Barlow's (1958) demonstrations.

Jones (1959) and Barlow (1962a) reexamined the question of quantum efficiency, and feeling uncomfortable with the assumptions Rose (1948) made to determine his "quantum efficiency of the eye", decided to measure instead the quantum efficiency of the whole observer. They provided the first assumption-free measurements of the quantum efficiency of the observer.[20] That much is not controversial. The interesting part is that Jones and Barlow suggested that they were measuring the same thing as Rose, but by a more reliable technique, when in fact they were measuring something fundamentally different.

Here is Jones's (1959) approach:

> "In 1946, Rose published a monumental paper in which he described a new concept, here called 'detective quantum efficiency,' that described the performance of the human eye in terms permitting direct comparison

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[20] Except, as Barlow (1962a) pointed out, Jones (1959) estimated pupil sizes from Reeves's (1920) tables, and while the observer used two eyes, Jones allotted the ideal the light corresponding to only one eye. The latter is a trifling matter; allowing the ideal to have the light corresponding to both eyes only entails halving the efficiencies Jones stated.

with nonbiological detectors, such as television camera tubes and photographic films.

". . . For physical detectors, the detective quantum efficiency is defined simply as the square of the ratio of (1) the signal-to-noise ratio in the output, to (2) the signal-to-noise ratio in the incident radiation

$$Q = \frac{(S/N)^2_{output}}{(s/n)^2_{photons}}$$

[I have substituted $(s/n)^2_{photons}$ for Jones's equivalent expression "$\overline{M}_s^2/\overline{M}_b$".]

"But with the detection process of human vision, the output of the observer is not a signal-to-noise ratio, but is rather a probability of detecting a given radiation signal. The output is thus the result of a decision-making process.

"In order to define the detective quantum efficiency of human vision, we must compare the decision-making ability of a human observer with the decision-making ability of an ideal decision-making device."

The last two paragraphs express Jones's idea, not Rose's. Rose referred to the "eye" and the "performance of the eye", not "human vision" and the "output of the observer". Jones (1959) continues,

"Suppose that to make a given kind of discrimination an ideal device would require the signal-to-noise ratio of the radiation to be at least as great as the number _k_ [i.e. d']. . . . Then the detective quantum efficiency of human vision is defined by

$$Q = \frac{k^2}{(s/n)^2_{photons}}$$

[Again I have substituted $(s/n)^2_{photons}$ for Jones's "$\overline{M}_s^2/\overline{M}_b$".]

"Rose, in all of his publications on this subject, has assumed that under the conditions of Blackwell's [1946] measurements the threshold signal-to-noise ratio was _k_=5, whereas in this paper we employ the much lower value _k_=1.22. This difference corresponds to a factor of 16.8 in the detective quantum efficiency. Rose's value is based on measurements of the threshold of signal of grainy photographic images and in noisy television images. . . . But the Blackwell data (. . .) used in Rose's calculations involve a condition (. . .) for which Table [of d' versus proportion correct] indicates _k_=1.52. Thus we conclude that the value of _k_ used by Rose is much larger than is appropriate for the performance of an ideal device.

"The present paper is the first in which the value of _k_ used in calculating the detective quantum efficiency of vision is based on the performance of the ideal device. The calculation of the values of k (for an ideal device) that are reported in Sec. 3 was made in response to a September 18, 1957, letter from Dr. H.B. Barlow, and I feel much indebted to him for emphasizing the need for basing the calculation on the value of k appropriate to an ideal detecting device, instead of on the signal-to-noise ratio required by the eye for some other visual task."

Jones is correct in saying that the signal-to-noise ratio of Blackwell's observer's performance (i.e. d') was k=1.52, but Rose's k=5 was meant to describe the threshold signal-to-noise ratio of the output of the eye, not the observer. Both Rose and Jones call k the "threshold signal-to-noise ratio", but Jones means d' and calculates it from the observer's performance at threshold, while Rose meant effective signal-to-noise ratio at threshold, and calculated it from thresholds in luminance noise.

Barlow (1962a) started with the implicit assumption that Rose's "quantum efficiency of the eye" is the same as the quantum efficiency of the observer and found contradictions which caused him to reject Rose's conclusions:

"Historical background. Quantum efficiency was proposed by Rose (1942) as an absolute measure of performance of an optical task and was later applied by him to the human eye (Rose, 1948). Two facts seemed to emerge at once from the use of this measure: first, the efficiency appeared to be remarkably high, suggesting that little loss of efficiency occurred except from the absorption of light in the optic media, and the failure to absorb all of it in the receptor cells of the retina: and secondly this high efficiency was thought to be maintained under a great diversity of working conditions of the eye. At this stage it seemed possible that much of the empirical psychophysical data on the visual performance could be summarized by the single statement that the quantum efficiency was high and almost constant (as de Vries suggested in 1943), but in fact both these early results have proved misleading. Aguilar & Stiles (1954) made a critical estimate of efficiency under conditions where Rose thought it was high, and obtained a lower value which decreased rapidly with increasing background intensity. Barlow (1958) showed that the range of conditions for constant quantum efficiency was very restricted; there is, for instance, no range of values of background intensity, and area and duration of an added increment, over which the values of threshold are consistent with the quantum efficiency for detection of the increment remaining invariant. Clark Jones (1957, 1959), pursuing Rose's line of thought, has obtained lower values of quantum efficiency, and has shown up more variation with variation of the experimental parameters."

Logically, at this juncture Jones and Barlow should only have concluded that the combination of Rose's two assertions was wrong. If Rose were wrong about k's constancy, the "quantum efficiency of the eye" might still be constant. However Jones and Barlow do not seem to have recognized that Rose's (1948) derivation (unlike Rose's, 1942, and de Vries's, 1943) of the Rose-de Vries law rested on two distinct and testable assumptions (that the transduction and calculation efficiencies are both constant).

The problem, I think, stems from a prejudice that Rose, Jones, and Barlow all shared. They all acknowledged the logical possibility of other sorts of inefficiency than unabsorbed photons, but at some crucial point all assumed that beyond the eye we are perfect. Rose assumed that threshold always occurred at the same signal-to-noise ratio, k, at the eye's output. Thus he thought that if one knew the "quantum efficiency of the eye" one could explain all thresholds, because the brain counts perfectly. Jones (1959) and Barlow (1962a) equated the signal-to-noise ratio at the observer's output, d', with Rose's k which represented the signal-to-noise ratio at the output of the eye. That implies a calculation efficiency of 1. The implicit assumption by everyone concerned that the observer's brain made perfect calculations implied that the quantum efficiency of the observer and the "quantum efficiency of the eye" should be equal. When it was found that the quantum efficiency of the observer was always less than the "quantum efficiency of the eye", the latter measure was discredited. None of these papers seriously considered that it might be possible to separately measure the efficiencies of the "eye" and the "brain" (see Figure 2.2).

Barlow (1977) has recently noted that the highest estimates of quantum efficiency in which one may have full confidence are about 5%, while the lowest photometric estimate of the fraction of corneal photons that excite rods is about 11%. He attributed the discrepancy to a "central efficiency" of 50%. As Figure 2.2 shows, Barlow's "central efficiency" is the calculation efficiency. He said,

> "The current line of thinking makes one doubt the assumption that the cortex deals efficiently with the evidence provided to it by the sense organs - an assumption that has been implicit in much of the work on threshold. There is a method whereby this efficiency can be measured and this will now be described."

He went on to measure efficiency for detection of a small square increment in a static dot display, and found it to be about 50%. Here is Barlow's description of how he measured "central efficiency":

> "In these experiments the subject looks at an oscilloscope screen on which dots appear at random in controllable density and frequency of occurrence. They are, however, bright enough and few enough in number for it to be reasonable to assume that retinal factors do not limit their perception. . . . One hopes with such a stimulus to break through the retinal barrier and study the ability of central mechanisms to detect these patterns."

Barlow defined "central efficiency" as the detective efficiency in high noise, which equals our calculation efficiency. He correctly suggested that this might help explain the discrepancy between quantum efficiency and the photometric estimates of retinal absorption. However he did not take the next step of recognizing that the quotient of the quantum efficiency and the "central efficiency" was Rose's "quantum efficiency of the eye", even though he discussed van Meeteren and Boogaard's (1973) determination of that last quantity in the same paper. In the review of effects of luminance noise we will see that many people have measured efficiencies in high levels of luminance noise, which we may take as estimates of calculation efficiency, but only Barlow (1977) noted its relevance to quantum efficiency, which is determined in the absence of luminance noise.

METHODS

Sinusoidal gratings were displayed using a television-like raster on a cathode-ray tube by the method of Schade (1956) as modified by Robson (Campbell and Green 1965). The contrast of the grating was reduced toward the edge of the screen. The envelopes in the horizontal and vertical directions were approximately Gaussian. The screen was 15° (30 cm) wide and 10° (20 cm) high, surrounded by a much larger field of approximately the same hue and luminance. Room lights were left on; no direct illumination fell on the display. The screen displayed 100 frames per second, otherwise the space-average luminance, L_{mean}, was constant throughout each experiment.

No head restraint was used but the viewing distance, 114cm, was sufficiently large that postural motions of the observer (at most ±5cm) could change the spatial frequencies by at most ±5%. The observer viewed the screen with both eyes and natural pupils. All of the observers wore their optical corrections, if any. DP was corrected for the 114 cm viewing distance with +.25 diopter (right eye) and -.25 diopter with -.5 diopter cylinder at 60° (left eye).

Figure 2.4 shows the exact luminance of the display (in cd/m^2), and an estimate of the retinal illuminance (in trolands) based on measurements of the observer's pupil size two years after the experiment was done, under similar conditions.

Every figure has an inset which describes the characteristics of the signal and noise used in that particular experiment. The bottom line of the inset specifies the space-average luminance (L_{mean}), and a code number. The second two letters of the code number are the initials of the observer's first and last names. Following that are identifying numbers assigned to the blocks in which the the data were collected.

Signal

The inset includes a "signal" box in which may appear a sinusoid with a Gaussian envelope, indicating that the signal was a grating vignetted in space and faded on and off in time by multiplication of the signal and Gaussian envelopes. The spatial frequency of the grating is specified. Signal contrast is defined in the absence of noise as $(L_{max}-L_{min})/(2L_{mean})$. The extent of each Gaussian is specified by its 1/e points, i.e. its horizontal space constant ("x"), its vertical space constant ("y"), and its time constant ("t"). Note that the full width, height, or duration at more than 1/e of peak amplitude would be twice the value specified.

Noise

Pseudorandom noise was digitally synthesized by a 31-bit shift register with exclusive-or feedback. On each shift the Boolean "exclusive or" of bits 13 and 31 was shifted into bit 1. The cycle repeated after every $2^{31}-1 \approx 10^{10}$ shifts. To produce one-dimensional noise, as in Figure 2.1 there was one shift per raster line. At this rate (100kHz) the cycle would have repeated twice a day. To produce two-dimensional noise, as in Figures 2.3 and 2.4, the spatial-frequency bandwidth of the noise was maximized by outputting samples as fast as possible: 10 MHz. At 10 MHz the cycle repeated every 7 minutes. The noise was free-running, not synchronized to the frames of the display or signal presentations, making it very unlikely that the observer could benefit by any memory of the previous cycle, seven minutes ago. Each pseudorandom bit produced one sample. Prior to attenuation each sample was either +2.5 volts or -2.5 volts. The samples were uncorrelated, so the noise spectrum was nearly flat up to 5 MHz, half the sample rate.

The noise was attenuated by a passive attenuator (75Ω L-pad sections) controlled by the computer (using relays). The attenuator directly drove a coaxial cable with a 75Ω characteristic impedance terminated in its characteristic impedance at the display, where it was added to the signal. This arrangement resulted in a very flat transfer function; the gain was down by only 1 dB at 10 MHz. Capacitive feedthrough of the attenuator was negligible. I am grateful to J.G.Robson for suggesting the construction of a 75Ω attenuator as a way of achieving the needed bandwidth.

The inset includes a "noise" box with a snapshot of the noise, as it might appear at one moment. The noise was always dynamic, never static. Two-dimensional, dynamic, white noise looks like shifting sand. One-dimensional, dynamic, white noise looks like hanging strings fluttering in the wind.

The white noise spectrum is described by its spectral density. Because the displayed noise is made up of discrete samples (i.e. frames temporally, (vertical) raster lines horizontally, and the pseudorandom samples vertically) the spectrum is white, that is, flat and uncorrelated, up to the Nyquist frequency: half the sample frequency f_s in each dimension. The "noise" box in the figure inset lists the Nyquist frequencies in each dimension. The horizontal Nyquist frequency appears below the picture, the vertical Nyquist frequency appears to the right of the picture, and the temporal Nyquist frequency appears above the picture. Thus the listed frequencies describe the band over which the noise was white.

Display linearity

The display exhibited no significant nonlinearity up to contrasts of 90%. The extended range of linear operation was possible because the luminance modulation circuit, designed by J.G.Robson, sensed the cathode current in order to enforce a linear relation between input voltage and beam current. The linearity was checked at the beginning of every experimental session by the simple expedient of counterphase flickering a sinusoidal grating at a rate above critical flicker fusion, and confirming that the display appeared uniform. Second harmonic distortion would produce a static grating of twice the original spatial frequency. This method has been independently devised by Bernice Rogowitz and Jacob Nachmias (personal communication).

The Michelson contrast of the noise was always kept sufficiently low so that the sum of signal and noise was within the linear regime of the display.

Procedure

Two-interval forced-choice (2IFC) was used. The observer initiated each trial. Each interval was marked by an auditory tone and lasted 300 msec. The signal appeared randomly in one or the other of the two intervals. Its envelope reached its peak halfway through the particular interval. The noise was always on and was at constant strength throughout the whole trial.

A PDP-11/20 computer synthesized the signal and ran all the trials. Threshold was measured by the method of two-interval forced-choice. Usually several conditions (noise levels) were interleaved. The noise was always on the display; before each trial the computer adjusted the attenuation of the noise to bring it to the desired spectral density (which could be zero) for that trial. Then the observer initiated the trial by pressing a button. Two intervals followed, marked by tones. The signal was presented during only one of the intervals, pseudo-randomly determined. Then the observer indicated, by button push, which interval he thought contained the signal. Finally a feedback tone indicated whether his response was correct.

It well known that trials using signal contrasts near the threshold will contribute most towards an accurate estimate of threshold (Levitt 1971). Therefore a method of data collection was devised which presented each signal at the current maximum-likelihood estimate of threshold for that condition. Similar methods have been described previously by Hall (1968) and Pentland (1980). It is efficient; 60 trials provide precise threshold estimates (standard deviation usually less than 12%, i.e. 1 dB). While the procedure did provide a threshold estimate, since it is relatively new (and thus to some extent untried) the data (# right and wrong at each contrast) were stored on disk for later analysis.

The stored data were reanalyzed by making a maximum likelihood fit with a two-parameter ($c_{.82}$ and β) Weibull (1951) function,

$$P(c) = 1 - .5e^{-(c/c_{.82})^{\beta}},$$

similar to that proposed by Quick (1974), using the computer program QUICK, as described by Watson (1979). In addition to the threshold estimate $c_{.82}$ (contrast required for 82% correct) it also provides a measure of the goodness of fit (2 times the log of the likelihood ratio of the best fit of the Quick model to an unconstrained psychometric function). It was determined empirically that when this index of likelihood was less than -15 the threshold estimate was unreliable. Only the threshold estimates with likelihood index above this criterion are reported. For this reason, about one in twenty threshold estimates was rejected for experienced observers. For novice observers about half of the first two or three threshold estimates were rejected by this rule. Each threshold estimate not rejected by this rule is plotted as a point on the graphs.

Each point on the graphs, unless otherwise indicated, is the maximum likelihood estimate of threshold (82% correct), based on the observer's responses to 60 trials. The reader may estimate the variability by noting the small scatter among the plotted points. d' was 1.27, i.e. $\sqrt{2}$ times the normal deviate corresponding to the proportion correct, 82%.

Each block consisted of several, typically four, interleaved conditions. Usually each condition used a different noise level. 60 trials were collected for each condition, but the conditions were randomly interleaved.

Since the noise level was very high in some conditions (sufficient to raise threshold more than tenfold) there was initially some concern that the threshold elevation might persist onto subsequent lower-noise-level conditions. However no systematic difference was found between zero-noise thresholds measured in a block containing only zero-noise conditions, and zero-noise thresholds measured in a block where the other three conditions were high-noise conditions, even though the procedure guaranteed that the observer was exposed to the high noise level about three-quarters of the time.

Calculating photon noise

In all the calculations in the Results and Review sections, retinal illuminance is expressed as Trolands (pupil area in mm^2 times the luminance in cd/m^2), and then converted to photon flux, $J_{photons}$, or photon noise,

$$n^2_{photons}=1/J_{photons}.$$

However because the photon flux and photon noise are desired solely for calculation of efficiencies, the values I give for these quantities will reflect all photons, whether they are distributed among both eyes or all sent into one eye.

RESULTS

Effect of noise spectral density

Figure 2.1 shows contrast thresholds as a function of the spectral density of one-dimensional, dynamic, white noise. One-dimensional, dynamic noise, as indicated in the inset, varies only in the horizontal and temporal dimensions: it formed fluctuating vertical lines. Threshold is plotted as a function of the noise spectral density for two spatial frequencies, 1 and 8 c/deg. The noise had little effect up to the critical spectral density, $n^2_{critical}$, indicated by the dashed line. Above it the squared contrast threshold is approximately proportional to the spectral density of the noise. The thresholds are well fit by the smooth curves representing proportionality of squared threshold to sum of actual and critical spectral densities of the noise. The fit was made by eye. Recalling that we named the sum of actual- and critical- the "effective"-noise level, the smooth curves describe proportionality of squared threshold to effective spectral density of the noise,

$$c^2 \propto n^2_{critical} + n^2.$$

Also shown are straight lines, the asymptotes for actual noise levels approaching zero (the horizontal line) and infinity (the unity-slope line). The asymptotes intersect at the "critical" spectral density of the noise.

Inter-observer variability

Figure 2.3 shows five observers' thresholds for detection of a 3 c/deg grating with and without a noise background. The noise background raised threshold by more than a factor of ten. The observers all gave very similar results. The standard deviation across observers was about 13% in both

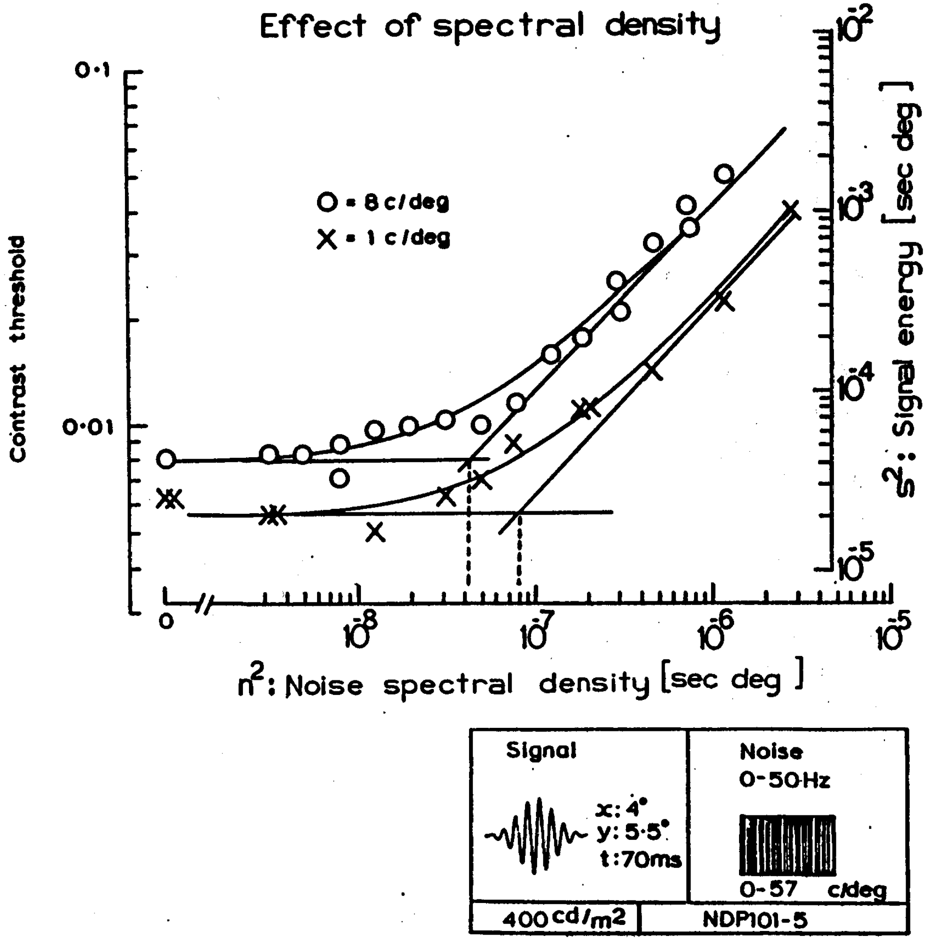
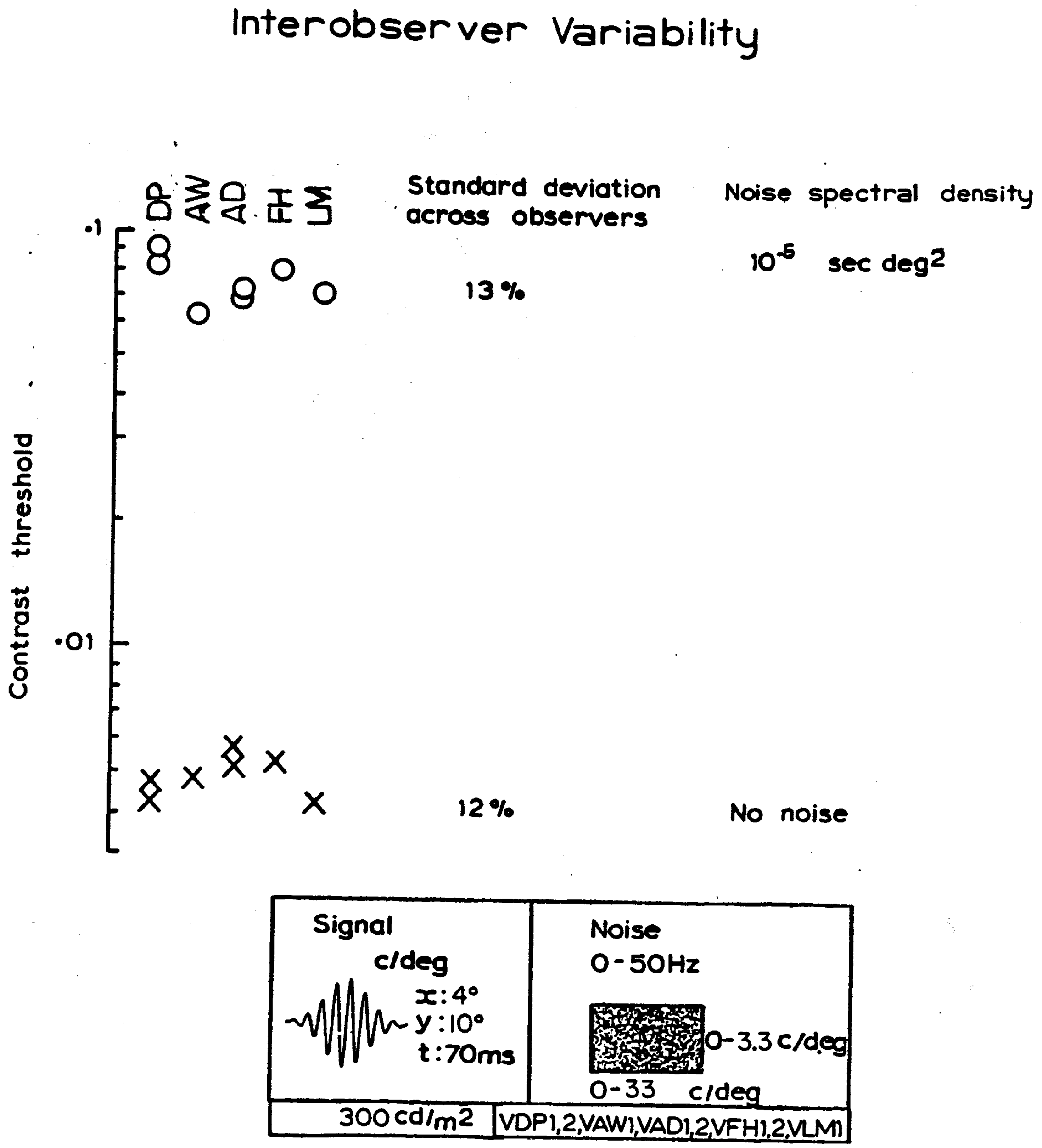

Effect of spectral density
0·1
0·01
Contrast threshold
O = 8 c/deg
X = 1 c/deg
0
10^{-8}
10^{-7}
10^{-6}
10^{-5}
n^2: Noise spectral density [sec deg]
10^{-2}
10^{-3}
10^{-4}
10^{-5}
s^2: Signal energy [sec deg]
Signal
x: 4°
y: 5·5°
t: 70ms
Noise
0-50Hz
0-57 c/deg
400 cd/m2
NDP101-5

Figure 2.1

Figure 2.3


Interobserver Variability
Contrast threshold
·1
·01
DP
AW
AD
FH
LM
Standard deviation across observers
Noise spectral density
13 %
10^{-5} sec deg²
12 %
No noise
Signal
c/deg
x: 4°
y: 10°
t: 70ms
Noise
0-50Hz
0-3.3 c/deg
0-33 c/deg
300 cd/m2
VDP1,2,VAW1,VAD1,2,VFH1,2,VLM1

conditions, only slightly more than the standard deviation of a single observer. The variations from the mean are consistent for each observer, but are not obviously correlated between the two conditions. One of the observers (FH) had never before acted as a psychophysical subject, others (DP and AW) were very experienced. At the time of this experiment DP had been observing gratings in noise almost daily for six months. None of the other observers had ever made observations in noise before. The similar thresholds of all the observers seems to rule out any large learning effect.

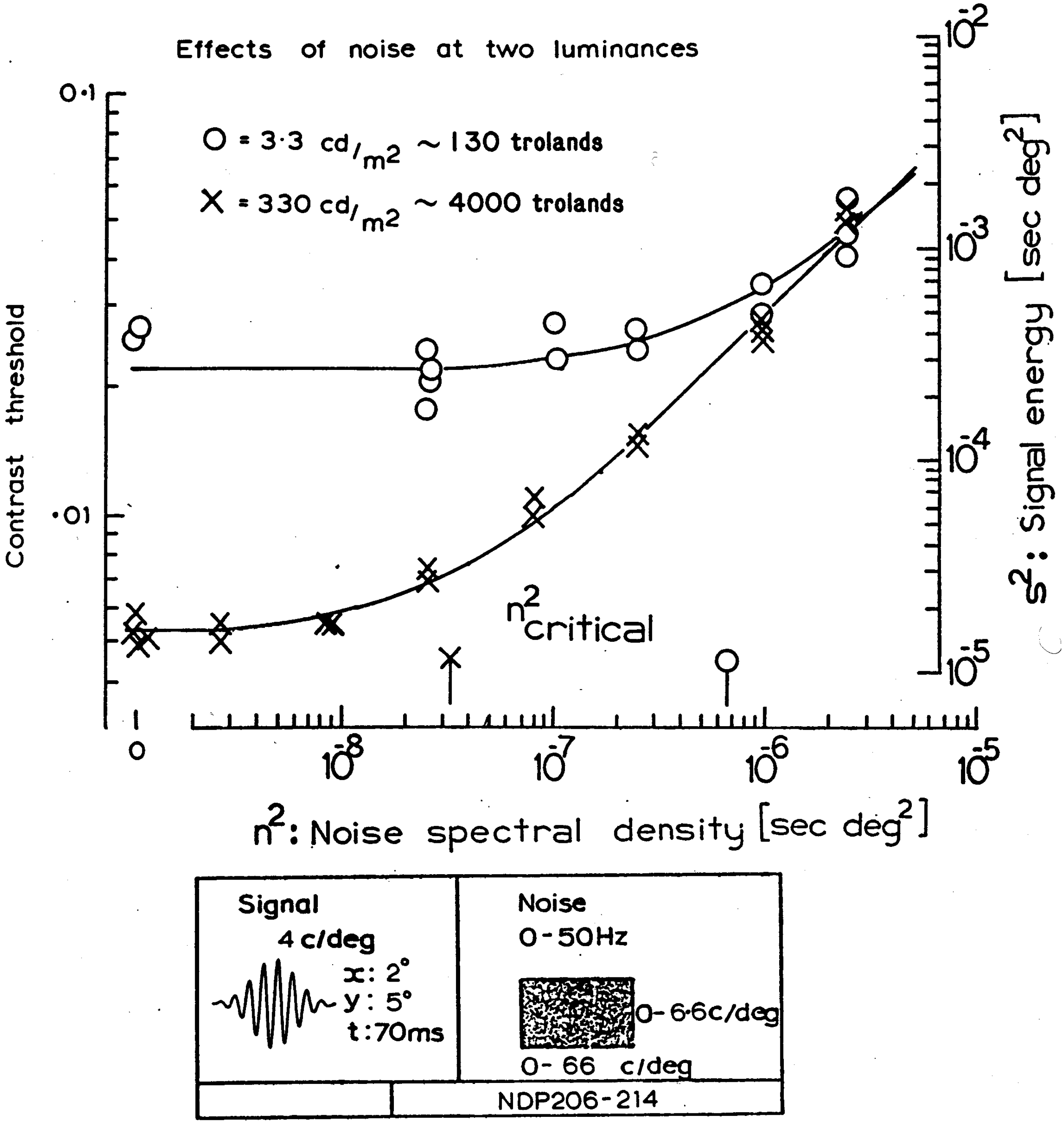
Most of the threshold measurements reported here were made only on DP, the author. The 2IFC method was used to avoid effects of criterion changes and preconceptions on threshold estimates. While it would be worthwhile to confirm some of the findings on other observers, the striking similarities found here suggest that little difference will be found between observers.

Effects of noise at two luminances

Figure 2.4 plots threshold for a 4 c/deg grating as a function of noise level at two luminances: 330 cd/m^2 and 3.3 cd/m^2. The noise was white, two-dimensional, and dynamic. The thresholds at each luminance are well fit by smooth curves representing proportionality of squared threshold to actual plus critical spectral density of the noise. Note that the curves converge at high noise levels, indicating little or no change in calculation efficiency. In the absence of noise, lowering the luminance by two orders of magnitude increased threshold in zero noise by a factor of four, but thresholds in high noise were unaffected.

The equation of the smooth curves is

Figure 2.4



Effects of noise at two luminances
O = 3·3 cd/m2 ~ 130 trolands
X = 330 cd/m2 ~ 4000 trolands
0·1
·01
Contrast threshold
n2 critical
0
10-8
10-7
10-6
10-5
n2: Noise spectral density [sec deg2]
10-2
10-3
10-4
10-5
s2: Signal energy [sec deg2]
Signal
4 c/deg
x: 2°
y: 5°
t: 70ms
Noise
0-50Hz
0-6·6c/deg
0-66 c/deg
NDP206-214

$$c^2 \propto n^2_{critical} + n^2.$$

The fitted values of $n^2_{critical}$ are indicated on the abscissa.

We have seen that squared threshold is proportional to the sum of the critical and actual noise levels. Lowering the luminance from 330 to 3.3 cd/m^2 increased the critical spectral density, without changing the constant of proportionality, implying that the calculation efficiency was constant. With all parameters shown explicitly, our relation becomes

$$s^2 = \frac{(d')^2}{CE} (n^2_{critical} + n^2).$$

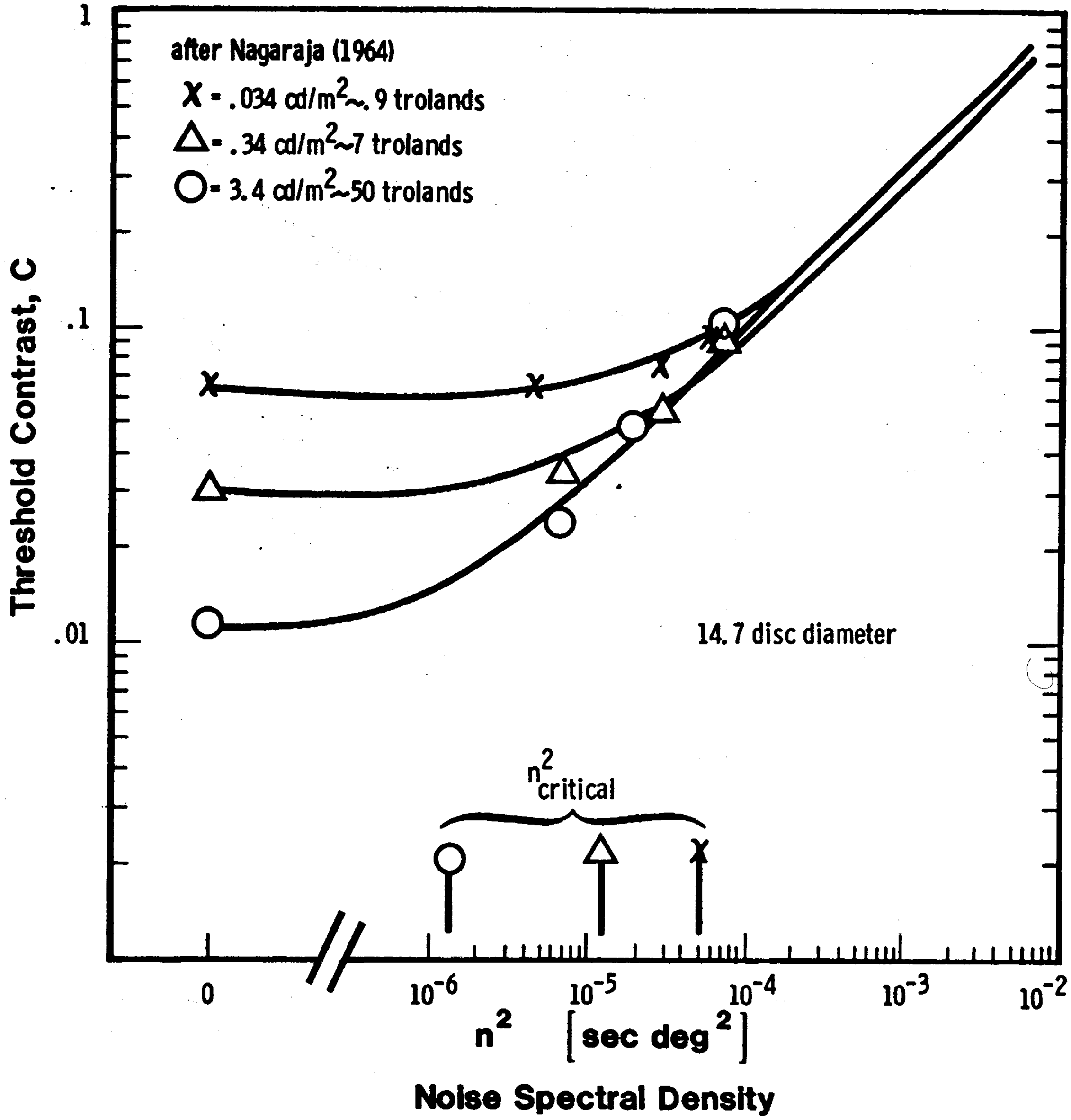
d' was 1.27 throughout, corresponding to 82% correct in the 2IFC task. The fitted curves indicate that the calculation efficiency was .4% at all noise levels and at both luminances. The critical spectral density was 2.3 10^{-8} sec deg^2 at 330 cd/m^2 (approximately 4000 trolands), increasing to 6.5 10^{-7} sec deg^2 at 3.3 cd/m^2 (approximately 130 trolands).

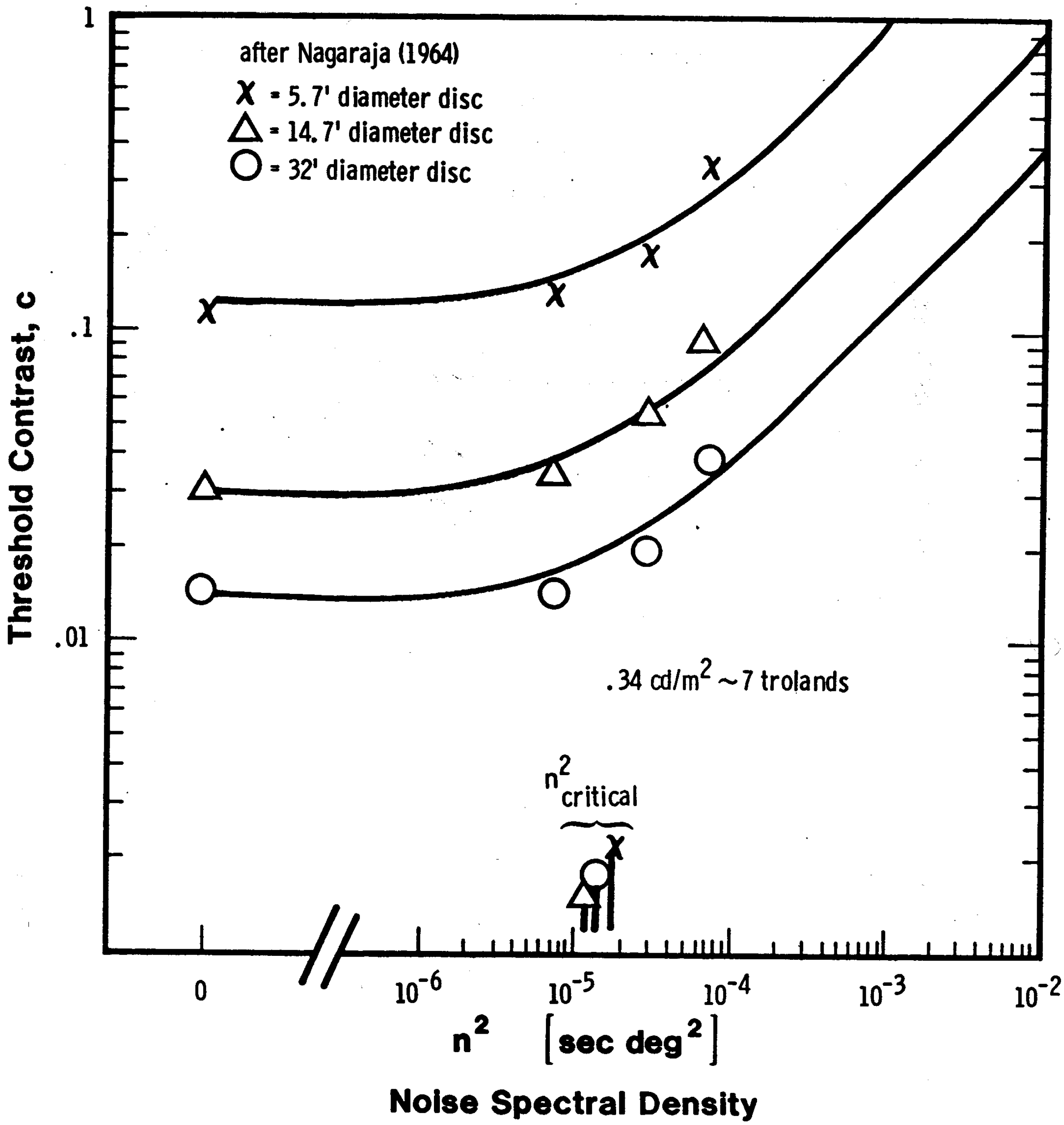
Estimates of transduction efficiency from published work

Figures 2.5 and 2.6 show similar data from Nagaraja (1964). Figure 2.5 shows contrast thresholds for a 14.7' diameter disc as a function of noise level, at three luminances, .034, .34, and 3.4 cd/m^2, corresponding approximately to .9, 7, and 50 trolands respectively. As before curves representing proportionality of squared threshold and effective noise level were fit by eye, and the critical spectral density esimates thus obtained are indicated on the abscissa.

Figure 2.6 shows threshold for three different disc diameters (5.7', 14.7', and 32') as a function of noise level at one luminance (.34 cd/m^2 = 7

Figure 2.5


after Nagaraja (1964)
X = .034 cd/m2 ~ .9 trolands
△ = .34 cd/m2 ~ 7 trolands
○ = 3.4 cd/m2 ~ 50 trolands
Threshold Contrast, C
1
.1
.01
14.7 disc diameter
n2 critical
0
10-6
10-5
10-4
10-3
10-2
n2 [sec deg2]
Noise Spectral Density


after Nagaraja (1964)
X = 5.7' diameter disc
△ = 14.7' diameter disc
○ = 32' diameter disc
.34 cd/m2 ~ 7 trolands
n2 critical
Threshold Contrast, c
1
.1
.01
0
10-6
10-5
10-4
10-3
10-2
n2 [sec deg2]
Noise Spectral Density

trolands). Note that the three disc sizes result in virtually identical estimates of $n^2_{critical}$.

Figures 2.7 and 2.8 show results from dot experiments of van Meeteren and Boogaard (1973) and van Meeteren (1973). Figure 2.7 shows threshold contrast for a 4.5 c/deg grating, 3°×3° viewed foveally, as a function of noise level, at three retinal illuminances (.035, .35, and 3.5 trolands). Figure 2.8 shows similar results for a 2.2 c/deg grating viewed 7° in the nasal field, at five retinal illuminances (.0013, .013, .13, 1.3, and 13 trolands).

Transduction efficiency versus retinal illuminance

Finally, the estimates of $n^2_{critical}$ from the preceding graphs (Figures 2.4 through 2.8) were used to calculate the transduction efficiencies which are plotted in Figure 2.9 as a function of retinal illuminance. The highest of the transduction efficiency estimates is 2.6% at .0016 troland, 7° nasal field, and the lowest is .43% at 4000 trolands. Unlike quantum efficiency estimates where sometimes only the values for optimal signals are reported, Figure 2.9 shows transduction efficiency for all data sets that could be found which report thresholds with and without two-dimensional dynamic white noise at the same luminance. These estimates vary less than tenfold over nearly seven decades of light level, and a variety of signals.

The sloping dashed line represents the highest quantum efficiencies Barlow (1962b) found, while the horizontal dashed line represents Barlow's (1977) lower bound (11%) on the photometric estimate of the fraction of corneal photons that excite rods.

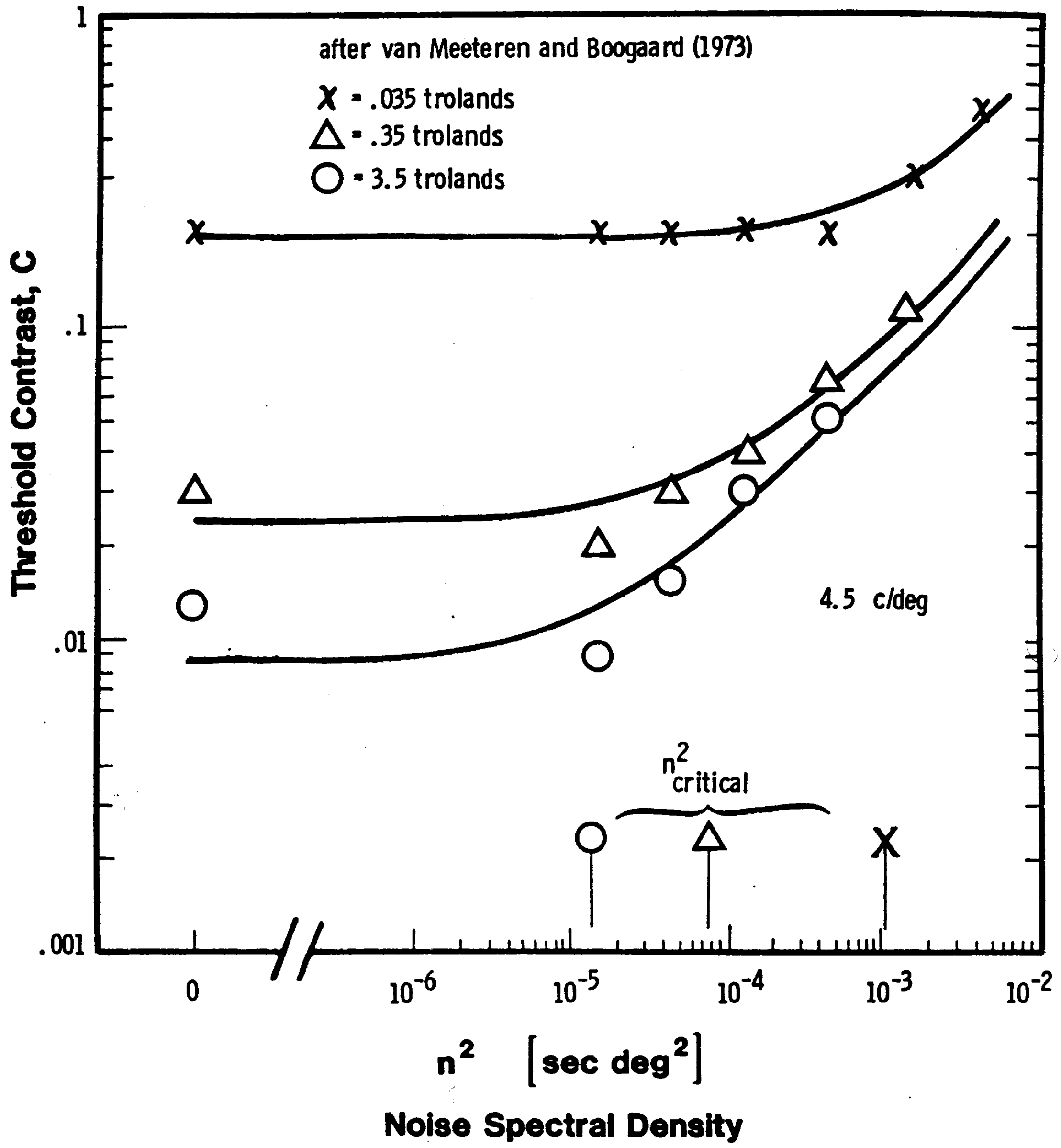
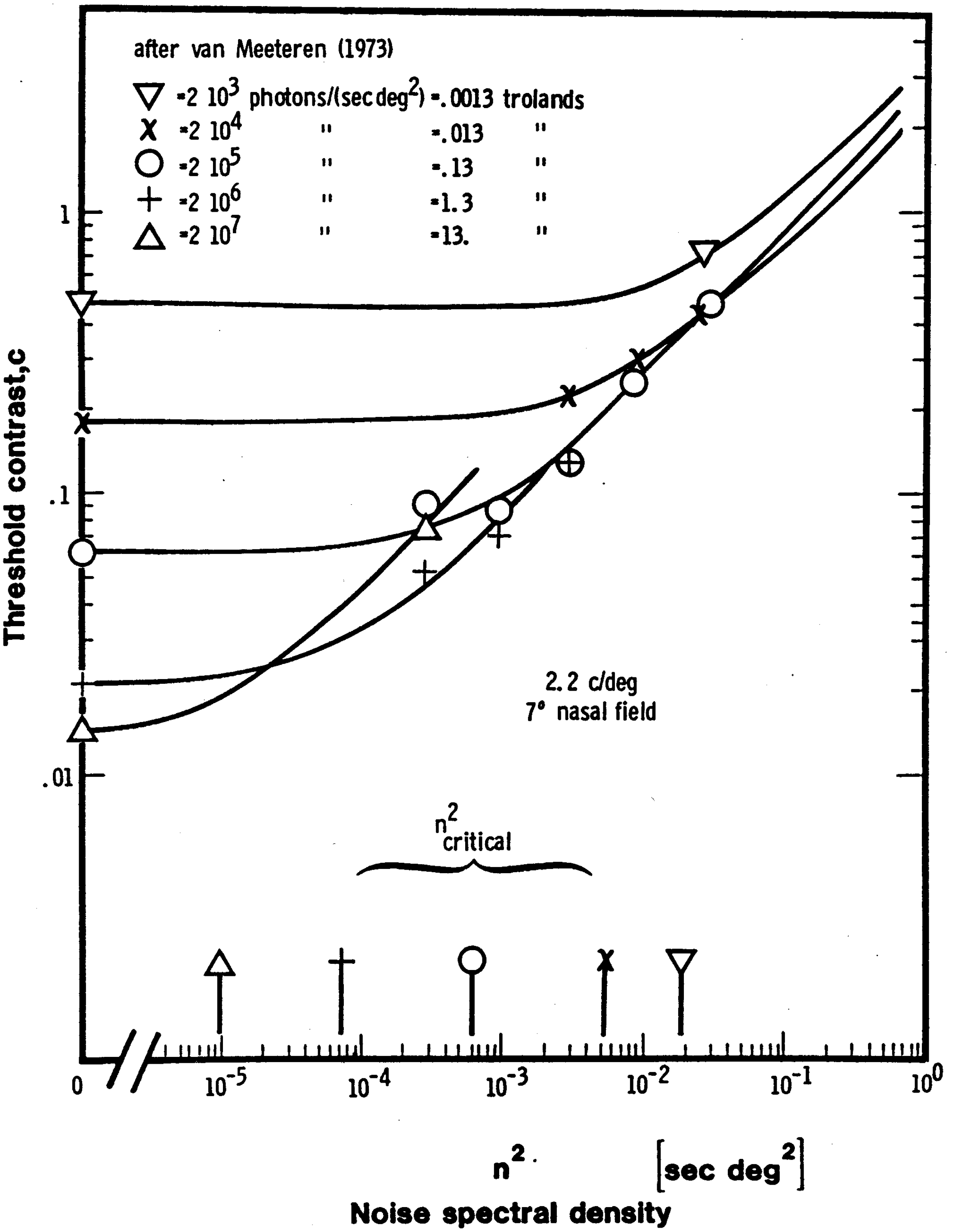
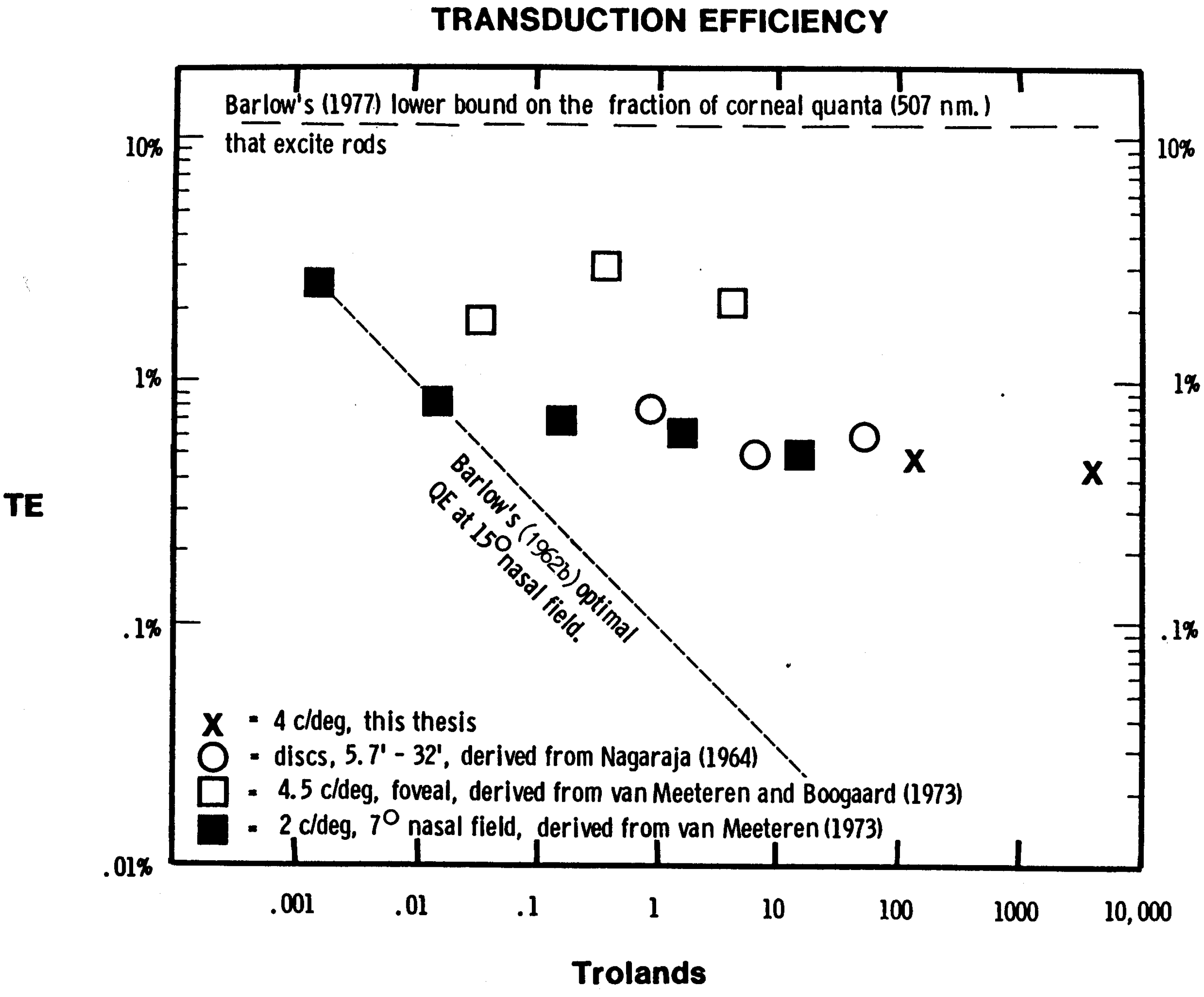

after van Meeteren and Boogaard (1973)
X = .035 trolands
△ = .35 trolands
○ = 3.5 trolands
4.5 c/deg
$n^2_{critical}$
Threshold Contrast, C
1
.1
.01
.001
0
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
n^2 [sec deg^2]
Noise Spectral Density

Figure 2.8



after van Meeteren (1973)
▽ =2 10^3 photons/(sec deg^2) =.0013 trolands
X =2 10^4 " =.013 "
O =2 10^5 " =.13 "
+ =2 10^6 " =1.3 "
△ =2 10^7 " =13. "
Threshold contrast,c
1
.1
.01
2.2 c/deg
7° nasal field
n^2 critical
0
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
n^2 . [sec deg^2]
Noise spectral density



TRANSDUCTION EFFICIENCY
Barlow's (1977) lower bound on the fraction of corneal quanta (507 nm.) that excite rods
10%
1%
.1%
.01%
TE
Barlow's (1962b) optimal QE at 15° nasal field.
X = 4 c/deg, this thesis
○ = discs, 5.7' - 32', derived from Nagaraja (1964)
□ = 4.5 c/deg, foveal, derived from van Meeteren and Boogaard (1973)
■ = 2 c/deg, 7° nasal field, derived from van Meeteren (1973)
.001
.01
.1
1
10
100
1000
10,000
Trolands

Figure 2.9

REVIEW

Thresholds in white noise

Rose's (1942, 1948) and de Vries's (1943) best-known contribution is now called the Rose-de Vries law. For Rose, at least, it rested on two assumptions: (1) that "quantum efficiency of the eye" (i.e. transduction efficiency) is constant,

$$TE = \text{constant},$$

and (2) that threshold occurs at a given "threshold signal-to-noise ratio", k, at the output of the eye,

$$k = \text{constant}.$$

We have seen that Rose's k is the effective signal-to-noise ratio, i.e. square root of the ratio of signal energy, s^2, to effective noise, $n^2_{effective}$,

$$k \equiv (s/n)_{effective} = s/\sqrt{(n^2+n^2_{critical})}.$$

So Rose's second assumption is,

$$(s/n)_{effective} = \text{constant}.$$

If the thresholds all have the same d', as in the Blackwell (1946) data Rose used, then this assumption implies that the calculation efficiency is constant. Assuming constant transduction and calculation efficiencies implies a constant quantum efficiency. The Rose-de Vries law is just the statement that the quantum efficiency is constant.

Rose tested his assumption that k, the effective signal-to-noise ratio, is constant at threshold by measuring thresholds for discs on a dot display. He implemented an ideal image intensifier which produced a bright dot for each detected photon. Imaging a test pattern of discs of various areas and contrasts, Rose found that the product of threshold contrast and disc diameter was constant, as predicted. This was also true for static photographs of the display. In these experiments Rose found that the effective signal-to-noise ratio at threshold was 5. By assuming that threshold is always at an effective signal-to-noise ratio of 5, Rose could estimate the fraction of the corneal quanta which were transduced by the eye. Rose was the first to compare effects of photon noise and luminance noise. Rose's paper was seminal for the study of both phenomena, yet since Rose they have usually been treated as unrelated.

Sturm and Morgan (1949) applied Rose's (1948) ideas to roentgenoscopy (now called fluoroscopy: examination of a patient's x-ray shadow by direct viewing of a fluorescent screen), and roentgenography (now called radiography: a photographic plate is placed in contact with the fluorescent screen, and after development the resulting "radiograph" is examined). It was known that with similar x-ray fluxes radiologists could make out much more detail in a radiograph than by fluoroscopy. Many suspected that this difference was due to the high luminance at which the radiographs were viewed (approximately 300 cd/m^2) and the low luminance of the fluoroscopic screen (.001 to .01 cd/m^2). Apparently it was thought at the time that the visibility of detail on the fluoroscope would at least equal that in radiographs if the fluoroscope screen could be made sufficiently bright (by increasing the number of visible photons emitted per incident x-ray photon). Sturm and Morgan used Rose's fluctuation theory to show that the greater visibility of detail in the radiographs was primarily due to the use of film exposure times much longer than our eye's integration time, so that the radiologist effectively used more of the x-ray quanta when he viewed the radiograph.

Sturm and Morgan made a transparency of an array of discs of graded area and contrast and held it against the scintillating fluoroscopic screen. They determined contrast thresholds for each disc size. They used Rose's estimates of the "quantum efficiency of the eye" (5%), and "integration time" (.2 sec), and the threshold signal-to-noise ratio (5) to predict the value of the product $c\surd(AI)$, where c is contrast, A is disc area, and I is screen luminance. The contrast thresholds for the discs were as predicted, except that the smallest and largest discs had higher thresholds. This showed that fluoroscopy was approximately light-limited.

They also, in effect, modulated the x-ray beam by an array of discs, made a radiograph, and then determined contrast thresholds from the developed radiograph. Again they used fluctuation theory to predict the product $c\surd(AI)$, where I is now dot flux (i.e. detected x-ray photons per second per screen area). Again the thresholds were higher than predicted for the smallest and largest discs. This confirmed that radiography was approximately x-ray-exposure limited.

Their most important contribution was the idea that thresholds for dot images would be limited by the total noise whose power equals the sum of the power at the retina of the photon noise and the dot noise, and that one or the other would usually be sufficiently greater to make the smaller one negligible.

> "Now these sources of fluctuation in the roentgenoscopic screen process are essentially independent of one another . . . Therefore, it may easily be shown by statistical theory that the average total fluctuation at the termination of the roentgenoscopic process when the nerve impulses are finally transmitted from the retina to the brain is given by the equation
>
> $$s^2=s_1^2+s_2^2+ \ . \ . \ . \ +s_n^2$$
>
> where $s_1, s_2, \ . \ . \ . \ , s_n$ are the values at the retina of the various average fluctuations introduced at the various phases of the roentgenoscopic process."

The practical conclusion of their paper was that increasing the brightness of the fluoroscopic screen (without increasing the detected x-ray flux) would only reduce the photon noise (i.e. inverse of effective photon flux) which was not much larger than the dot noise (i.e. inverse of detected x-ray photon flux), so that even a much brighter screen could not be as useful as the radiographs.

Schade (1951, 1952, 1953, 1955) did signal-to-noise ratio analyses of photographic and television systems. He later said (Schade 1964),

> "I was told by Dr. Gardner of NBS [National Bureau of Standards] at one of those lectures, I believe it was in 1952, that the great interest aroused by these lectures in the optical and photographic professions was due to the fact that I demonstrated for the first time that sine-wave response and noise in televison and photographic systems could be evaluated on a unified basis and measured with practical instruments, yielding objective data on the performance of practical system elements and complete systems. Without such instrumentation and numerical evidence, there would have been little interest in the mathematical theory."

Coltman (1954) used an ideal image intensifier to image a ten-bar pattern (i.e. 10 cycles of a square wave) of adjustable contrast. Squared threshold (50% "seen") was approximately proportional to the reciprocal of speck flux (i.e. proportional to spectral density) over two orders of magnitude.

Green, Wolf, and White (1959) used a computer to generate still photographs made up of a matrix of black and white dots. Each dot had a prior probability of being white (versus black); patterns were created by modulating the prior probabilities. Using this static dot display they presented 4 cycles of a square wave (with one probability over the lighter bars, and another, lower, probability over the darker bars). They measured the detectability of these square waves by asking the observer to state whether the bars were horizontal or vertical. They determined the signal contrast[21] required for

75%-correct responses by the observer.

> "To a first approximation the threshold is directly proportional to the standard deviation of the dot-density distributions. . . . The threshold of the ideal statistical detector has the same properties as the thresholds obtained . . . , but is much lower. Thus we may say that for a fixed pattern, and a fixed exposure, our observers behave like the ideal detector, except for sensitivity."

In other words, the efficiency was constant.

Coltman and Anderson (1960) measured contrast threshold (50% "yes") for a sinusoidal grating on a television display with added two-dimensional dynamic white noise. They found an initial fall in threshold over the first five trials of 3 dB (i.e. to .7 of the initial value). This probably reflects a lowering of the criterion for "yes" by the observer. "The effect of changing viewing distance over a range of 30:1 is hardly outside the experimental error in determining the threshold." Since changing the viewing distance doesn't change the signal-to-noise ratio, s/n (because s^2 and $\bar{n}^2$ are reduced equally by increasing the viewing distance), this implies a constant signal-to-noise ratio at threshold for a 30:1 range of spatial frequencies. And so they concluded,

> "It is found both theoretically [with fluctuation theory] and experimentally that the masking effect of white noise depends only on the noise power per unit bandwidth, and is independent of the upper frequency limit of the noise spectrum, provided that this exceeds the frequency limit set by the eye."

Griffiths and Nagaraja (1963) and Nagaraja (1964) reported thresholds (50% correct, after correction for guessing) for bright discs masked by white noise (dynamic, and two dimensional). The target was a bright disc, 6', 15', or 32' in diameter, which appeared for 8 seconds in any of 8 possible locations.

Nagaraja found that the square of the threshold contrast was proportional

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[21] Their definition of threshold wasn't contrast; it was the difference in prior probability between the white and black bars, but, for any given mean luminance the two measures are proportional and the meaning of their quoted conclusion is unchanged.

to the sum of the spectral density of the noise plus a constant, the "internal noise" (see Figures 2.5 and 2.6). This is evidence that calculation efficiency is independent of noise level. The "internal noise" was equal, within experimental error, for the three targets (see Figure 2.6). The "internal noise" was higher at lower luminances, and he showed that the "internal noise" was of similar magnitude to that expected from fluctuations in the number of quantal absorptions in the retina at the three luminances tested (.03, .3, and 3 cd/m^2), assuming .5% to 1% absorption.

Nagaraja (1964) calculated that his "intrinsic noise" might be due to photon noise, taking what he felt was a reasonable guess as to the fraction usefully absorbed. It is more interesting to do the opposite: calculate what fraction of the photons would have to be usefully absorbed to account for his "intrinsic noise", i.e. transduction efficiency. I calculate[22] Nagaraja's transduction efficiencies to be 0.8%, 0.5%, and 0.6% at estimated retinal illuminances of .9, 7, and 50 trolands, respectively. They are plotted on Figure 2.9. The differences between these transduction efficiencies are not significant because the pupil diameters were not measured, but only estimated later from standard tables, and thus the pupil areas may be in error by a factor of two in either direction (e.g. see Wyzecki and Stiles p. 213).

Griffiths and Nagaraja (1963) compared the human performance data with ideal performance. They present graphs of proportion correct in the eight-alternative forced choice task as a function of the signal-to-noise

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[22] Pupil sizes were not reported so the retinal illuminance in trolands was estimated directly from the luminance using a standard graph (Wyzecki and Stiles 1967, p. 214). Since the observers used both eyes the total photon flux $J_{photons}$ was twice that for one eye.

ratio. They reported that the human observer required a signal-to-noise ratio almost three[23] times greater than the ideal. For an 8 second presentation (8AFC) efficiencies (in luminance noise) calculated from their data are 4% for a 5.7' diameter disc, 7% for 9.8', 6% for 14.7', 11% for 21', and 8% for 32'. For brief presentation (2AFC) of the 21' disc the highest efficiency was 13% for a 320 millisecond duration.

Schade (1964) had observers identify the smallest visible bar triplet on a US Air Force 3-bar test object in the presence of a range of static white noise levels. He found the signal-to-noise ratio of these thresholds to be nearly constant over a wide range of noise levels.

Chambers and Courtney-Pratt (1969) asked observers to determine the location of a randomly-located patch of higher dot density in a background of randomly-located black dots on a white background. The patch could be anywhere on the square background. They compared the observers' performance with that of fluctuation theory, and, for good measure, a computer that acted as an ideal observer. The computer's performance was always very close to the theoretical prediction. Threshold was taken to be the number of extra dots in the signal patch required for 50% correct identification of location.

> "The computer when instructed to pick the signal location on the basis of a highest dot count, required about $\frac{2}{3}$ as much signal as a human observer for correct detection on 50% of the plots."

This implies a constant efficiency of $(\frac{2}{3})^2$=44%.

Pollehn and Roehrig (1970) displayed sinusoidal gratings with added noise on a television display. Some peculiarities in their results indicate that the modulation transfer function of the display was far from flat[24], but this would

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[23] The reader desiring to check this calculation should be aware of their unconventional definition of "dB, as 10 log c" (Nagaraja, 1964), whereas dB conventionally represent 20 log c.

not affect the results of interest to us now. They plotted threshold contrast, c, versus rms noise contrast for four spatial frequencies and fit each by a curve

$$c \propto \sqrt{(k^2 + c_{rms}^2)}$$

where c_{rms} was the rms contrast of the white noise added to the signal. As discussed in Appendix 1, for a given bandwidth, the spectral density, n^2, of white noise is proportional to its contrast power, c_{rms}^2. They only say that, "This fit implies an interpretation of [k] as the internal noise of the visual channel." The dependence of k on the spatial frequency of the signal is confounded by the MTF problem alluded to above.

Goodenough, Rossman, and Lusted (1972) carefully studied the detectability of a small spot of light "projected onto uniformly exposed radiographic samples exhibiting the coarse fluctuations in density, called radiographic mottle, which constitute a source of noise in the detection process." They asked

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[24] In their figure 11 they plot threshold contrasts for three spatial frequencies as a function of viewing distance. In effect this varied the screen size from 2 to 60 degrees. For 2 c/deg increasing the screen size by a factor of 4 increased sensitivity by a factor of 4. For 6 c/deg increasing screen size first increased, then decreased sensitivity. For 15 c/deg increasing screen size by a factor of 3 decreased sensitivity by a factor of 3. Effects of screen size have been reported by others, but they are always of the same form: a steep reduction in sensitivity when less than 4 periods are displayed (e.g. Campbell and Robson 1968), and a very gradual increase (a factor of 1.2 per octave) in sensitivity as more periods are displayed (Robson in Mostafavi and Sakrison 1976, Robson and Graham 1981). The contradiction may be resolved by supposing that the modulation transfer function (MTF) of the tv display used by Pollehn and Roehrig was far from flat. To test this theory I replotted their three graphs of sensitivity versus observing distance all on one graph of sensitivity versus number of bars across the screen. The three curves then differed only by a scale factor in sensitivity. This graph indicated that the MTF of the display had a full bandwidth of 4 octaves at half height, centered at about 80 cycles/screen. They state that, "The bandwidth of the apparatus was 8.5 MHz (3 dB point)." However, the MTF I calculated from their data has already fallen a factor of 4 (i.e. 12 dB) at 6.5 MHz. Since all the noise spectra were measured only at the input of the display, we cannot be sure of the spectra of the displayed patterns. This prevents quantitative interpretation of all of their results that involve more than one spatial frequency or viewing distance.

observers to rate (on a five point scale) the likelihood that the spot was present. The spot could be anywhere on the radiograph, estimated by the authors to include about 800 non-overlapping positions. Detectability of the signal on one noise background ("resulting from a fast speed screen - medium speed film combination") was much higher than on another ("resulting from a medium speed screen - fast speed film combination") which had approximately twice the spectral density and bandwidth as the first.

Stromeyer and Julesz (1972) measured threshold by the method-of-adjustment for a sinusoidal grating in one-dimensional dynamic white noise (white over 0-30Hz[25] and 0-54 c/deg). For two observers for three spatial frequencies, the square of the threshold was proportional to the spectral density[26] of the noise over the two orders of magnitude tested.

Rosell and Willson (1973) measured thresholds (32% correct, 4AFC) for bright rectangles presented for 10 seconds in dynamic two-dimensional white noise. With some assumptions[27] their results indicate that at a constant luminance and noise spectral density and threshold criterion, the observer's calculation efficiency seems to be constant for rectangles in the range .1°×.1° to .5°×4°, and about fourfold lower for rectangles with larger minimum dimensions (i.e. 1°×4° and 2°×2°).

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[25] Their 60 Hz frame rate implies that the noise spectrum is white only up to the Nyquist frequency, 30 Hz. This issue was discussed at the beginning of this chapter.

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[26] Stromeyer and Julesz did not report the spectral density, but it is easily calculated as the ratio of the power c_{rms}^2 and the (double-sided) bandwidth, which were both reported.

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[27] Results are reported in terms of the "signal-to-noise ratio" at threshold, but they do not clearly define the term. In one respect their quantity is certainly suspect: their calculations reduce the signal duration to .1 seconds, to account for the "integration time" of the observer's eye.

Image intensifiers were a natural application for fluctuation theory. However image intensifiers for field use have compromised a variety of desirable qualities, presumably in order to maximize portability and quantum efficiency. In particular the imaging properties are far from uniform over the field. Therefore experiments designed to study vision have used apparatus similar to that used by Rose (1948), a flying spot scanner linked to a kinescope (i.e. a closed-circuit television monitor). Quantum efficiencies tend to be low but plenty of light is available. Such a device will be called an ideal image intensifier, to distinguish it from the commercial image intensifiers designed for field use.

In a review of his earlier work, Rose (1957) said,

> "There are other more direct ways of measuring quantum efficiencies. These involve a simple side-by-side comparison of the human eye with other picture-recording devices of known quantum efficiency. For example, let an observer and a television camera view the same test pattern side-by-side. The observer need then only compare what he sees of the test pattern with what he sees reproduced on the television monitor to decide whether his quantum efficiency is greater or less than that of the television camera. The great virtue of this method is that one is not dependent on a knowledge of the storage time of the eye or of the threshold signal-to-noise ratio. Since the same observer compares the original and reproduced patterns, the same storage time and threshold signal-to-noise ratios are effective and their influence on the comparison cancels out."

Rose's estimate of the "quantum efficiency of the primary photo process" (i.e. the transduction efficiency) is the ratio of the number of dots for threshold with the image intensifier (at a high luminance) divided by the number of corneal photons for the same threshold contrast with unaided vision (at a much lower luminance),

$$TE = J_{dots}(\text{high luminance})/J_{photons}(\text{low luminance}).$$

This operational definition of the "quantum efficiency of the eye" differs slightly from Rose's (1948) earlier one, but insignificantly for our current purposes.[28] Rose (1957) suggested we calculate the "quantum efficiency of the eye" from the noise level at a high luminance that produces the same threshold contrast as zero noise at a lower luminance. Let us call that the <u>iso-threshold</u> <u>method</u>. We have based estimates of transduction efficiency on two different thresholds at the same luminance. Let us call it the <u>iso-luminance</u> <u>method</u>. Both methods depend on the calculation efficiency being equal for the two measurements. The iso-luminance method requires that the calculation efficiency be independent of the noise level, which available evidence indicates is true. The iso-threshold method requires the calculation efficiency be independent of luminance, but we will see evidence that this is often false. Nagaraja (1964) used the iso-luminance method, but most of the

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[28] We have seen that the 1948 definition is equivalent to the transduction efficiency. Rose's (1957) new technique is to measure how much luminance noise at a high luminance equates the photon noise at the low luminance. Because of this pure substitution of luminance noise for photon noise their ratio will not be affected by any component of the observer's equivalent noise that is independent of luminance. Thus this second definition of "quantum efficiency of the eye" would seem to be a purer measure of the efficacy with which photons are utilized. However, the method is impractical because few, if any, devices can be assumed to have a constant calculation efficiency over the wide range of luminances required.

If it is desired to leave out the luminance-independent part of the observer's equivalent noise it would be better to determine what that component is (by measuring $n^2_{critical}$ at many luminances) and subtracting it out before dividing into the photon noise. This might be called a "corrected transduction efficiency", "CTE",

$$CTE = \frac{n^2_{photons}(\text{at } L)}{n^2_{critical}(\text{at } L) - n^2_{critical}(\text{at } 100L)}.$$

In fact, over the ranges which $n^2_{critical}$ has been measured it falls so quickly with increase in luminance that the "corrected transduction efficiency" would not differ significantly from the standard transduction efficiency.

published estimates of transduction efficiency were made by the iso-threshold method and cannot be relied on.

Some experiments have been reported which used commercial image intensifiers (e.g. Ruedy in Jones 1959; Beurle 1969, van Meeteren, Vos and Boogaard 1971, Rosell, Svenson, and Willson 1972, and Engstrom 1974). Most of these were rough confirmation of the applicability of fluctuation theory to visibility of patterns in dot noise, but several went on to estimate transduction efficiency by the iso-threshold method. Jones (1959) reported,

> "In a February 13, 1959, letter, Dr.J.E.Ruedy reports an experiment in which a target at a luminance of 10^{-3} ft-L [.003 cd/m^2] was viewed directly, and also via a television system using an intensifier orthicon. Nine observers were used. It was found that when the detective quantum efficiency of the television system was ten percent, the detectability of the target was roughly the same by direct viewing and via the television system."

In a similar experiment using 533 nm monochromatic light, Engstrom (1974) reported transduction efficiencies between 1% and 4% (at 533 nm) from about .01 to 1 troland, except for one observer (the youngest at age 30) whose tranduction efficiency estimates were unaccountably nearly tenfold lower, in the range .2% to .5%.

van Meeteren and Boogaard (1973) built an ideal image intensifier like that used by Rose (1948); each detected photon produced a small brief speck on the display. They used it to image sinusoidal gratings. They defined threshold as the contrast required for 75% correct identification of the orientation of the grating which was always vertical or horizontal. At high speck energies (i.e. many photons per speck) threshold was independent of speck energy[29]. Using high-energy specks they confirmed earlier reports that contrast sensitivity was proportional to the square root of speck flux. That is, that squared contrast threshold was proportional to reciprocal speck flux, and thus proportional to spectral density,

$$c^2 \propto 1/J_{dots} = n^2.$$

They also reported measurements of contrast sensitivity for continuous gratings (i.e. not made up of specks) under otherwise identical conditions. They used Rose's (1957) iso-threshold method to calculate the transduction efficiency, which they called the fraction of quanta "effectively absorbed". Presuming that specks each containing 500 photons were fully effective, they determined the speck flux, J_{dots}, required to give the same threshold as obtained with a continuous grating at a lower luminance with photon flux $J_{photons}$. $J_{dots}/J_{photons}$ was 1% from 3.5 down to .35 trolands, and fell to .4% at .035 trolands.

Figure 2.7 shows their data replotted as threshold contrast versus noise spectral density. Each symbol represents data from one luminance, .035, .35, and 3.5 trolands. The data at each luminance are reasonably well fit by proportionality of squared contrast to effective noise level, except that for the two higher luminances threshold is higher without noise than with the lowest noise level. This is surprising. van Meeteren and Boogaard said,

> "The conclusion must be, that the particular stimulation with clusters of photons is more effective than the normal stimulation in this case."

Since the discrepancy is only a factor of 1.5, an alternate explanation is suggested by the fact that the zero-noise thresholds were collected as part of a previous study (van Meeteren and Vos 1972), and were not remeasured presumably because it was not feasable to produce zero-noise images with the

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[29] Their expectation was that with a sufficient number of photons per speck the observer would be able to utilize virtually every speck. This was confirmed; contrast sensitivity (i.e. 1/threshold) rose as the number of photons per speck (i.e. speck energy) was increased until a critical speck energy, beyond which threshold was constant. They found that the critical speck energy was not a constant, but depended on the luminance.

arrangement required for dot display.

The estimates of $n^2_{critical}$ were used to calculate transduction efficiencies (by the iso-luminance method) which are plotted on Figure 2.9 as open squares. The transduction efficiencies, 2.1%, 3%, and 1.8% at .035, .35, and 3.5 trolands, respectively, are higher[30] and more constant than van Meeteren and Boogaard's estimates (.4%, 1%, and 1.2%) calculated by the method of iso-threshold from the same data. As we noted earlier the iso-threshold estimates of transduction efficiency depend on the assumption that the calculation efficiency is independent of luminance. Figure 2.7 shows that the calculation efficiency for the 4.5 c/deg grating is virtually the same at .35 and 3.5 trolands, but is much lower (i.e. the high-noise threshold is higher) at .035 trolands. These data were all from a 3°×3° grating centered on the fovea. They showed that thresholds for frequencies in the range 2 to 15 c/deg were all raised in the same proportion in the speck image condition, relative to a noise-free condition, implying that the transduction efficiency is the same for 2 to 15 c/deg at these luminances.

In his thesis, van Meeteren (1973) includes the data of van Meeteren and Boogaard (1973) and similar data for a 2.2 c/deg grating at 7° nasal field as well. Figure 2.8 shows his data, replotted as threshold contrast versus noise

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[30] A small part of the reason my estimates are higher is that I assumed

$$1 \text{ lumen} = 4.1\ 10^{15} \text{ quanta (at 555 nm)/sec},$$

(Wyzecki and Stiles 1967) while van Meeteren and Boogaard seem to have assumed, for no apparent reason, that

$$1 \text{ lumen} = 5.3\ 10^{15} \text{ quanta/sec}.$$

I arrived at their figure by comparing the abscissas, one in cd/m^2 and the other in photons/(sec min^2 of arc), on otherwise identical graphs of zero-noise contrast sensitivity.

level, at five luminances (.0013, .013, .13, 1.3, and 13 trolands). The data are well fit by curves representing proportionality of squared contrast and effective noise level. The calculation efficiency is constant for the three middle light levels, and slightly lower (i.e. the high-noise threshold is higher) at the lowest (.0013 trolands) and highest (13 troland) light levels. It is to be expected that sensitivity (i.e. reciprocal of threshold) at a given speck flux would increase initially if speck energy (i.e. photons per speck) is increased. van Meeteren found that it

> "tends to decrease again when the speck-intensity is made very high, as was predicted by Beurle (1969) and demonstrated also by Hodgson[31]."

Beurle's prediction was based on consideration of changes in the eye's integration time and area with luminance, which is a possible explanation for the changes in calculation efficiency.

Again the estimates of $n^2_{critical}$ were used to calculate transduction efficiencies which are plotted in figure 2.9. The transduction efficiency is maximum, 2.6%, at .0016 troland; it falls gradually from .9% to .5% from .016 to 16 trolands. These transduction efficiency estimates seem very low. They are lower than the foveal transduction efficiencies at light levels (.016 to 1.6 trolands) which should favor rods over cones, and thus the periphery over the fovea. This may be due to my assuming that

$$1 \text{ lumen} = 4.1 \times 10^{15} \text{ quanta (555 nm)/sec}$$

which is, of course, inappropriate at these scotopic luminances; however, van Meeteren reported the light level only as photopic trolands, so there is no

[31] see Hodgson (1972).

obvious remedy. Given these uncertainties over photon fluxes there may be no significant difference between the approximately .4% transduction efficiency estimates from my data, the approximately .6% estimates from Nagaraja's data, and the approximately 2% transduction efficiency estimates from van Meeteren and Boogaard's data.

This agreement is particularly interesting since van Meeteren and Boogaard used sinusoidal gratings in speck noise, while Nagaraja used discs in continuous white noise, and I used gratings in continuous noise. All the transduction efficiencies on Figure 2.9 are within .4% to 3%. This includes discs of 5.7' to 32' in diameter, gratings of 2 to 15 c/deg, foveal and 7° nasal field viewing, and retinal illuminances of .0016 to 4000 trolands. The small variation of transduction efficiency may be contrasted with the enormous variation of quantum efficiency found over a comparable range. As shown in Figure 2.9, Barlow's optimal quantum efficiency (at 15° nasal field) fell two decades over the luminance range of van Meeteren's 7° nasal field data. For van Meeteren and Boogaard's data for 4.5 c/deg (Figure 2.7), the calculation efficiency changed fourfold while the transduction efficiency was approximately constant, indicating a fourfold variation of quantum efficiency where there is no change in transduction efficiency. van Meeteren and Boogaard showed that thresholds on their dot display were raised by the same proportion for all spatial frequencies from 2 to 15 c/deg, implying a constant transduction efficiency, but the signal energies at threshold indicate that quantum efficiency (and calculation efficiency) for 2 c/deg is 44 times higher than for 15 c/deg.

By its definition the transduction efficiency must exceed the quantum efficiency for the same signal. At the fovea, van Meeteren and Boogaard (1973) showed that gratings of 2 to 15 c/deg give the same transduction efficiency, so we might suppose a similar constancy in the periphery in order to allow a comparison with Barlow's (1962b) determinations of quantum efficiency. Barlow reported quantum efficiency for optimized targets at 15° nasal field. This is shown on Figure 2.9 as a dashed line with negative slope. However van Meeteren's data are from 7° nasal field. At 7° nasal field Barlow reported only the quantum efficiency in the dark. Barlow's quantum efficiency in the dark was higher at 15° nasal field (5%) than at 7° nasal field. Perhaps the transduction efficiency too is higher at 15° than at 7° nasal field.

Barlow (1978) reported efficiencies for detecting discs of various diameters and durations on a speck display. Barlow reported 8% efficiency for a 1 second 0.69° flash, which agrees well with Griffiths and Nagaraja's (1963) report of 6% efficiency for a 12' diameter disc exposed for 640 milliseconds on two-dimensional dynamic white noise. However with brief signals Barlow obtained much higher efficiencies than Griffiths and Nagaraja did. This is probably because Griffiths and Nagaraja kept the noise on continuously, while Barlow displayed specks on his display only during signal presentation. Barlow's brief signals resulted in a brief display, in effect, a glimpse of a static display. Elsewhere in the same paper Barlow showed that displaying a static display as long as the observer wishes instead of only 112 milliseconds improves his efficiency only "marginally".

With a static dot display (i.e. the prior probability of dots was higher in the target area, but the dots were otherwise randomly located) Barlow attempted a parametric study of the effects on detective efficiency of average background dot density, but this was confounded by changes in average luminance, as the luminous energy of each dot was kept constant. Nevertheless several of his results are noteworthy. When the background contained an average of only 8 dots, efficiency was 100%. This will be discussed shortly. Except for this unique condition, efficiencies were never significantly above 50%. Efficiency was about 50% for a 15' by 15' square target.

Chesters and Hay (1979a) measured threshold for detection of a static disc of light added to static two-dimensional noise. The discs were from .5' to 2° in diameter. The background was enlarged photographic grain, flat in spectrum up to 10 c/deg. Threshold was determined by increasing the signal contrast in steps until the subject said "yes". Catch trials measured the false positive rate (2.6% without noise, 4% with noise).

The contrast sensitivity function (i.e. reciprocal of threshold, as a function of disc diameter) was similarly shaped for the two observers with and without noise backgrounds. Squared threshold was proportional to the sum of the spectral density and a fitted constant. The fitted constant was interpreted as "intrinsic noise".

The lowest threshold signal-to-noise ratio was obtained for discs between 15' and 30' in diameter. Threshold rose for smaller and larger discs. Estimating d' in their experiments to be 1.95[32], and calculating the s/n to be 2.4 from their signal and noise measurements, the detective efficiency for the 15' to 30' discs was $(d')^2/(s/n)^2 \approx 66\%$. Because my estimate of d' is unreliable we can only conclude that the efficiency is not in disagreement with the approximately 50% value found by Barlow using briefly presented signal and

[32] They did not do a proper yes-no experiment, but they did state that "in a preliminary series of measurements no systematic difference was found between the 50% threshold obtained by the conventional [yes-no] method and by the modified version." Since they also reported the false-alarm rate, it was possible to estimate d' as the difference between the normal deviates corresponding to the 50% hit rate and the 2.6% false-alarm rate. Thus $d' = z(50\%)-z(2.6\%) = 1.95$.

noise.

Burgess, Humphrey, and Wagner (1979) measured method-of-limits thresholds for a bar optically superimposed on a background of static, two-dimensional noise,

> "the results are fitted by the expression
>
> $$c_{tN}^2 = c_{t0}^2 + kN$$
>
> where c_{t0} is the contrast threshold for no external noise, c_{tN} is the threshold for . . . external noise power N, and k is a constant."

<u>Dots and specks versus continuous noise</u>

Modulating dot rates is conceptually different from adding a continuous signal to a continuous noisy background, yet we have seen both experiments yield similar results when analyzed in terms of the squared threshold contrast of signal and the spectral density of the noise.

Mezrich (1979) provides some support for this view that dots are not special. Up to now we have only considered white dots on a black background. Mezrich showed that easily visible modulations become invisible if the contrast is reversed so that the dots are black on a white background. He noted that the (luminance) contrast of the expected signal was lower with the black dots. At a minimum these demonstrations show that black and white dots are not treated equally. Mezrich's approach merits quantitative assessment.

There is one respect in which the dot and speck displays seem to be special. Hirst, Beurle, Beverly, and Poole (1979) reported that at very low speck fluxes (less than 260 specks per $deg^2 sec$) observers' contrast thresholds are independent of speck flux and are much lower (i.e. more efficient) than predicted by extrapolation from higher speck fluxes. Barlow (1978) showed that observers detect targets on backgrounds with a mean of 8 dots with 100% efficiency, whereas efficiency was only 50% for 18 to 200 mean background dots.

Both papers suggested that the subjects might be "subitizing" (rapidly perceiving the number of a small group of objects) at the lower dot rates. We can accurately identify the number of randomly arranged dots in a briefly presented image containing up to about 4 or 5 randomly arranged dots (Jevons 1871, Kaufman, Lord, Reese, and Volkmann 1949). Jevons (1871) showed this by repeatedly throwing beans in a cup, glancing inside, and recording both an estimate and then the actual count. To accurately identify larger numbers we require more time and some method (e.g. eye fixations) to keep track of which dots have already been counted; Atkinson, Campbell, and Francis (1976) reported that,

> "the limit of 4 is [still] found if the stimulus is a bright afterimage lasting for approximately 60 seconds."

Barlow (1978) described the strategies of the observers for dot displays,

> "If the number . . . was under 7, it could be counted even in the brief exposure used in this experiment (Jevons 1871) . . . As the number increases one makes a 'numerosity judgement,' and it is the inaccuracy of this which causes efficiency to drop. At still higher numbers we thought that it was the texture, dot density, or average dot separation . . . , and above this again we thought subjectively that the mean brightness . . . gave the best cue."

However subitizing could not account for Barlow's data. Kaufman defined subitizing as the rapid perception of the number of 6 or fewer items, yet Barlow reported a 100% detective efficiency on a background with a mean of 8 dots. To attain the measured reliability ($d' \simeq 1$) the observer must have used a criterion of about $8+\sqrt{8} \simeq 11$ dots with few errors. This is much larger than the 4, 5, or perhaps 6, item limit which past authors have found for rapid, accurate assessment of numerosity, i.e. subitizing. Although estimation of more than 6 dots is inaccurate and has a longer reaction time, it would be accurate enough for the 11 item criterion implied by Barlow's result. Jevons's (1871) contingency table for reported versus actual bean counts indicates his reports had a standard deviation of about 1 when the actual count was 11. A dot display with a mean of 8 would have a variance of 8. The observer's responses would reflect the sum of the variances 8+1 = 9. Thus 8/9 = 89% of the variance underlying the observer's response would be due to the variance of the number of actually displayed, i.e. the efficiency would have been 89%. Barlow's 100% reported efficiency was not significantly different from 89%. To see how far this may be extended consider a 50 dot criterion. Jevons reported that "the average error of estimation . . . is simply proportional to the excess of the real number over $4\frac{1}{2}$," so extrapolating to 50 we estimate the standard deviation to be

$$(50-4\tfrac{1}{2})/(11-4\tfrac{1}{2}) \approx 7.$$

The variance of the stimulus would be 50, for an efficiency of $50/(50+7^2) = 51\%$. Barlow found "there is a range from 18 to 200 [mean background dots] where performance is about 50%." Thus Barlow's efficiencies for up to at least 50 background dots may be the result of numerosity estimation by the observer, but for parsimony we should remember that the constant efficiency, and thus a constant threshold signal-to-noise ratio also occur when the signal and noise are continuous, rather than made up of discrete items.

Hirst et al.'s suggestion of subitizing is more tenable[33], but estimation may be the correct explanation there too.

For backgrounds of 25 dots, Burgess and Barlow (1981) have recently shown that detection was not affected by approximately doubling the noise spectral density in a way that left dot density constant:

> "It is possible that there are cells in the visual system that respond to the total luminous flux emanating from the dots in one hemifield, and that estimation of dot number is based on the responses of such cells. If this were the case the accuracy of estimation would be seriously perturbed by brightening randomly selected dots in each hemifield. . . . the number of dots was judged equally well when some were dim and some 5 times more intense as when they were all at the some luminance."

Yet, in the same paper, Burgess and Barlow also report the effect of blurring the dot display. The observer viewed the display "through a diffusing screen placed at various distances from the oscilloscope screen." There was "little effect" even when the line spread function was as wide (at half height) as the entire display. The dot density was high relative to the (two-sided) bandwidth of the diffusing screen 2x1 x 2x1/screen, so that, with respect to the diffusing screen the display noise was white, and its Poisson origin would have been undetectable.

All together these data suggest that we adopt especially efficient strategies for static dot displays with less than perhaps 50 dots and dynamic dot displays with less than 260 dots/deg^{2}sec. At higher densities the discrete

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[33] Suppose that observers can use their subitizing ability to test whether or not a certain area and duration of the display contains more than 5 specks. Presumably the duration would correspond to the 100 msec or so apparent integration by the eye. Further suppose that the observer's criterion is (at least) two standard deviations above the mean (Hirst et al. measured threshold by the method of limits). Then the mean number of dots in the area-duration cannot exceed 2.1, with standard deviation $\sqrt{(2.1)} = 1.45$. This corresponds to a contrast of 150%, i.e. (criterion-mean)/mean. (Hirst et al. defined contrast of their discs as $\Delta L/L$, which can exceed 100%.) The hypothesized subitizer would be an ideal detector (i.e. 100% efficient) for targets with 5 or fewer specks (signal plus background). If there are more than 5 specks it would count over a smaller area-duration containing 5 specks so its contrast threshold will be constant at about 150%. In fact the thresholds in question were constant at about 100%, lower than predicted for a subitizer. In view of the speculative nature of this analysis it would appear that subitizing is a possible, if not convincing explanation of Hirst et al.'s data at low speck rates.

nature of these displays does not seem to affect detection; threshold seems to depend only on the expected signal and the spectrum of the noise.

CONCLUSIONS

Contrast thresholds in white noise have been found to obey the relation

$$c^2 \propto n^2_{critical} + n^2.$$

Dot experiments deviate from this relation only at dot fluxes and densities below that required for the assumption that the noise is white for the channel, and far below that required for the observer. On that basis it has been suggested that $n^2_{critical}$ is an estimate of the observer's equivalent input noise. The ratio of the photon noise to the critical spectral density has been called the transduction efficiency.

Since the observer acts in every way as though the critical spectral density were a luminance noise at his visual field, it has been suggested that the sum of that and any actual luminance noise be called the effective noise. Efficiency with respect to the effective noise has been called calculation efficiency. It has been found that the calculation efficiency is independent of noise level.

Perhaps the important contribution of this chapter has been to show that the quantum efficiency is the product of the transduction efficiency and the calculation efficiency. It is known that quantum efficiency depends strongly on parameters of the signal (Barlow 1958, Jones 1959). All the estimates of transduction efficiency that can be made (by the iso-luminance method) are between .4% and 3%. The conditions span nearly 7 decades of retinal illuminance, foveal and 7° nasal field viewing, and a wide range of patterns. This indicates that the violations of the Rose-de Vries law are due almost entirely to the dependence of calculation efficiency on signal and luminance.

The following chapters will include a few more considerations of the observer's equivalent noise, but are primarily concerned with calculation efficiency.

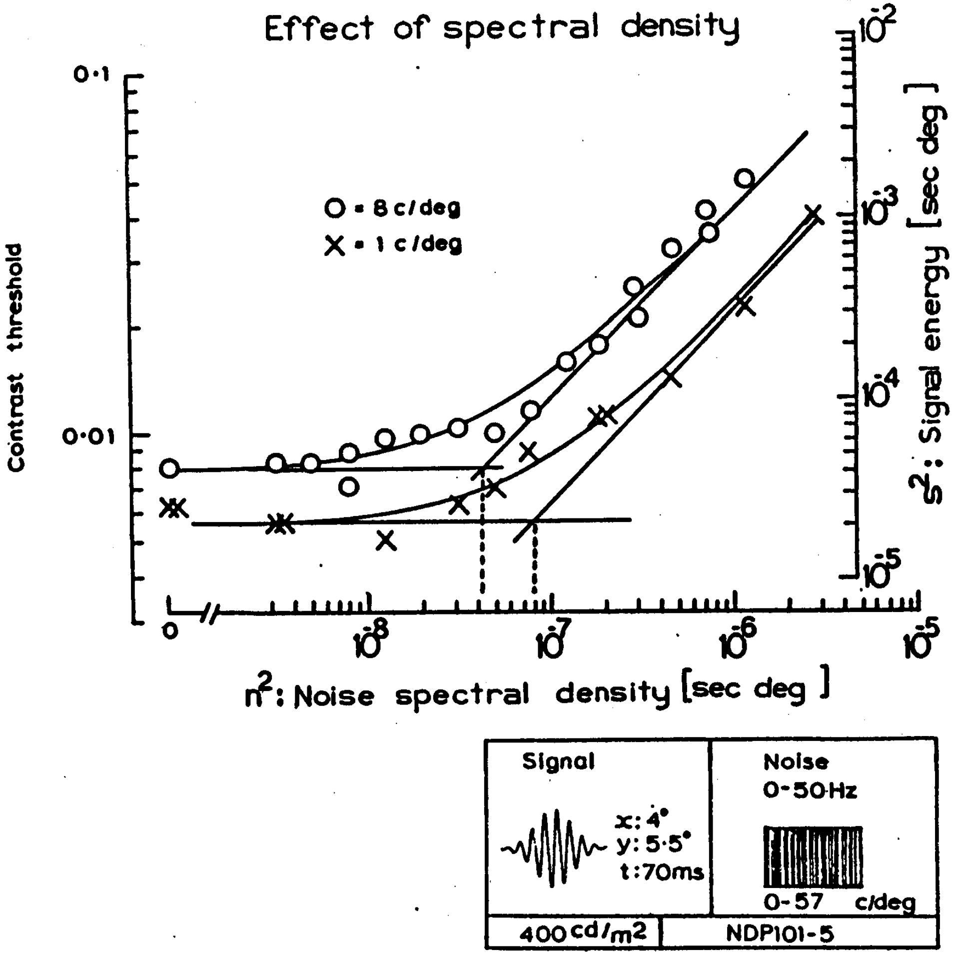
Transduction efficiency, like quantum efficiency, can be measured for any device, not just human vision. In photography it might help sort out different causes of inefficiency in the formation of an image. It seems particularly appropriate for recordings from visually-responsive cells, such as photoreceptors, retinal ganglion cells, and cells in the visual cortex. Initially it would be desirable to measure the contrast threshold in the presence of two-dimensional dynamic white noise to determine whether the relation

$$c^2 \propto n^2_{critical} + n^2$$

holds, supporting the interpretation of $n^2_{critical}$ as the cell's equivalent noise at the visual field, and confirming constancy of calculation efficiency. If that were found often enough to be assumed then the critical spectral density could be calculated from the contrast thresholds at any two noise levels, preferably zero noise and a high noise level, sufficient to raise the squared threshold by a factor of 2 or more. Any threshold criterion may be used.

Quantum efficiency, on the other hand, will be very low unless the signal is closely matched to the spatial and temporal properties of the cell. To obtain the highest quantum efficiency of which a cell is capable many signals must be tested (just as with the human observer), while the transduction efficiency may turn out to be the same for all signals to which the cell has any response.

It is quite possible that the equivalent noise (and the transduction efficiency) of cells at many levels in our visual system equals the equivalent noise (and the transduction efficiency) of the observer. However the transduction efficiencies of ganglion cells and the observer may be very different. For example, the transduction efficiency of a retinal ganglion cell could be half the observer's if each ganglion cell received input only from odd- or even-numbered receptors, but the information from all the ganglion cells were assembled more centrally. Alternatively, the transduction efficiency of retinal ganglion cells could be higher than that of the observer if later, more-central stages contributed a non-negligible amount of referable noise. Thus the transduction efficiency of a cell is an important clue in deciphering its connections to other cells, and the amount of information it is carrying.



Effect of spectral density
Contrast threshold
0·1
0·01
O = 8 c/deg
X = 1 c/deg
s^2: Signal energy [sec deg]
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
10^{-8}
10^{-7}
10^{-6}
10^{-5}
n^2: Noise spectral density [sec deg]
Signal
x: 4°
y: 5·5°
t: 70ms
Noise
0-50Hz
0-57 c/deg
400 cd/m2
NDP101-5

Figure 2.1

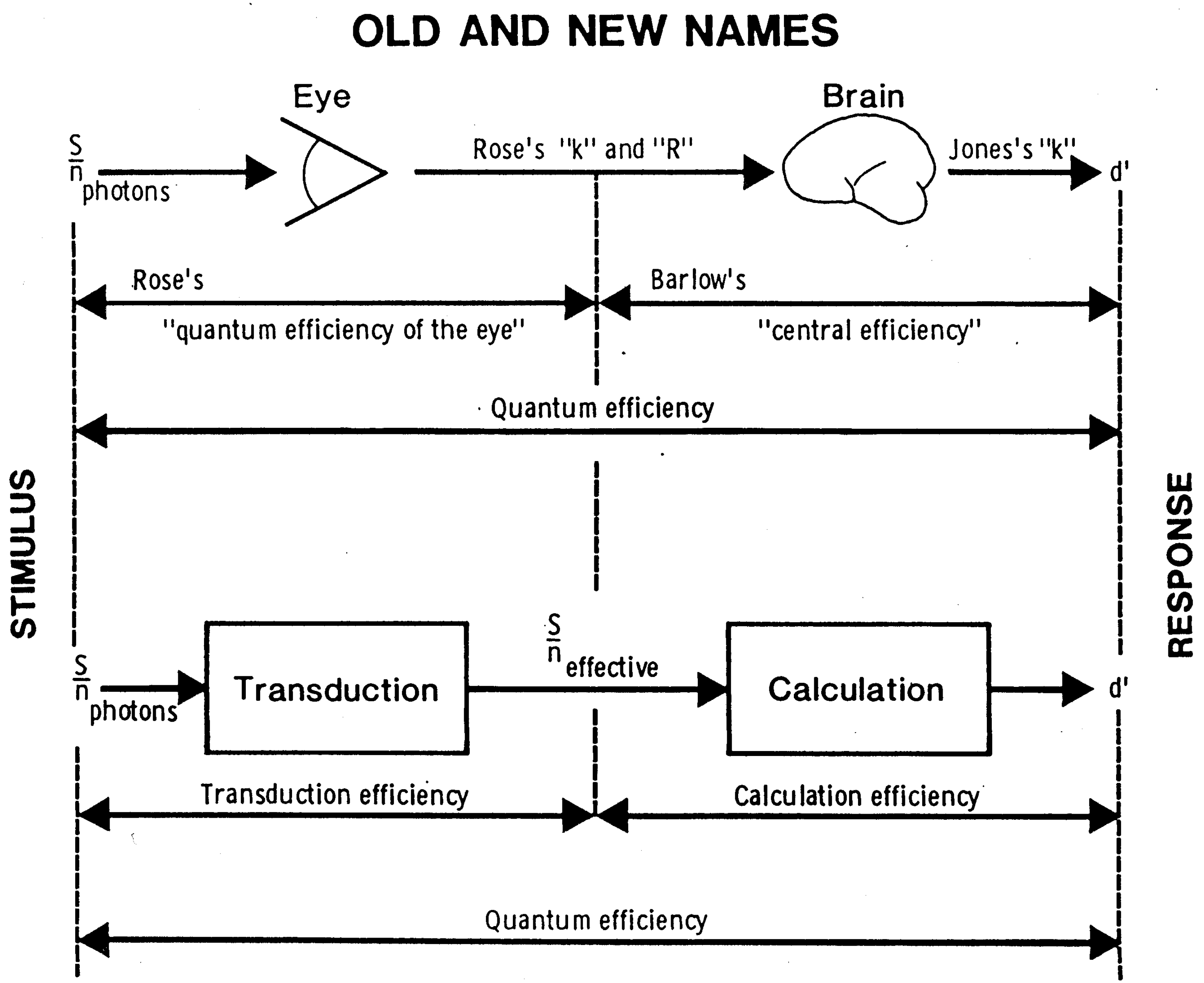
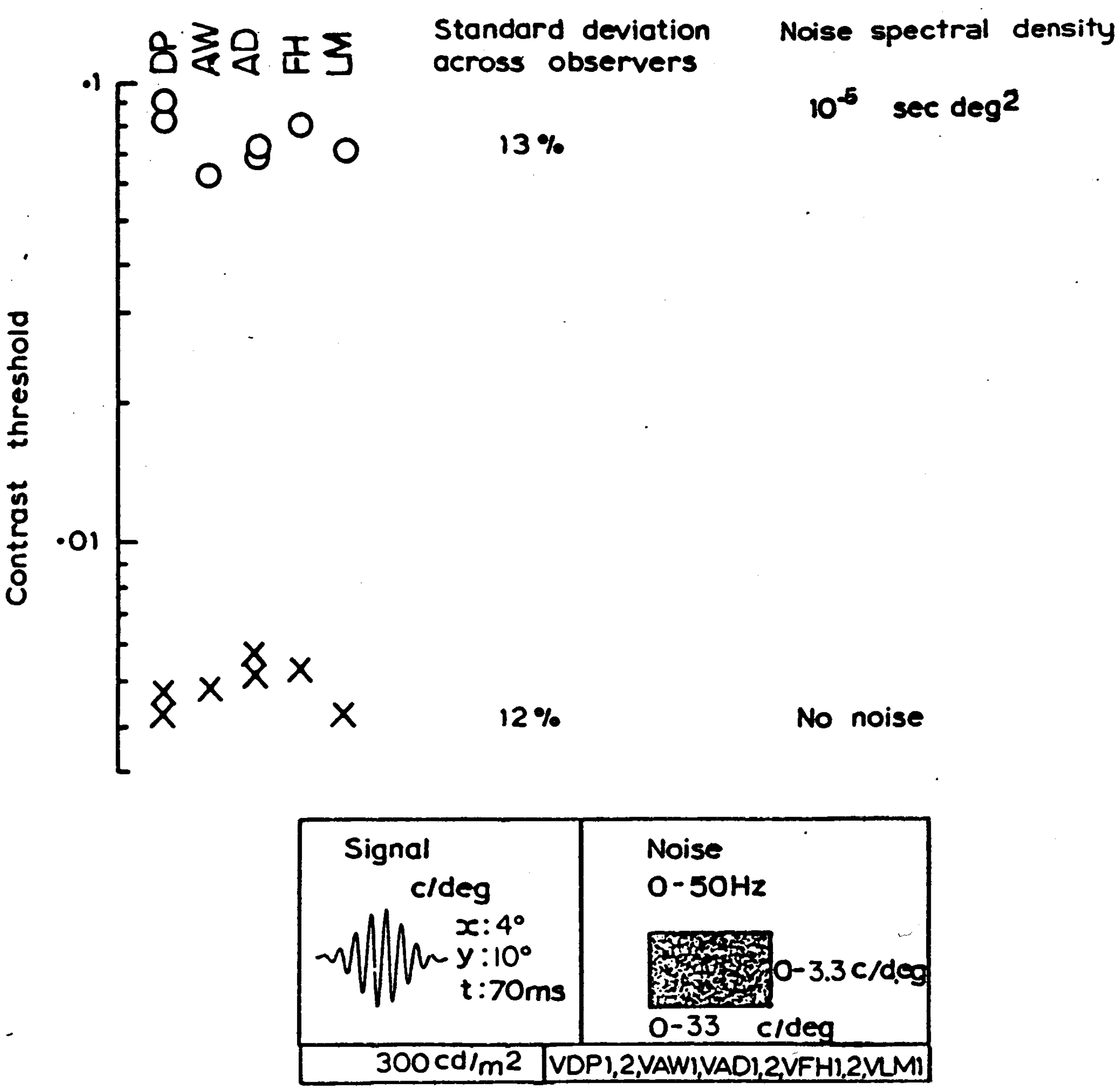
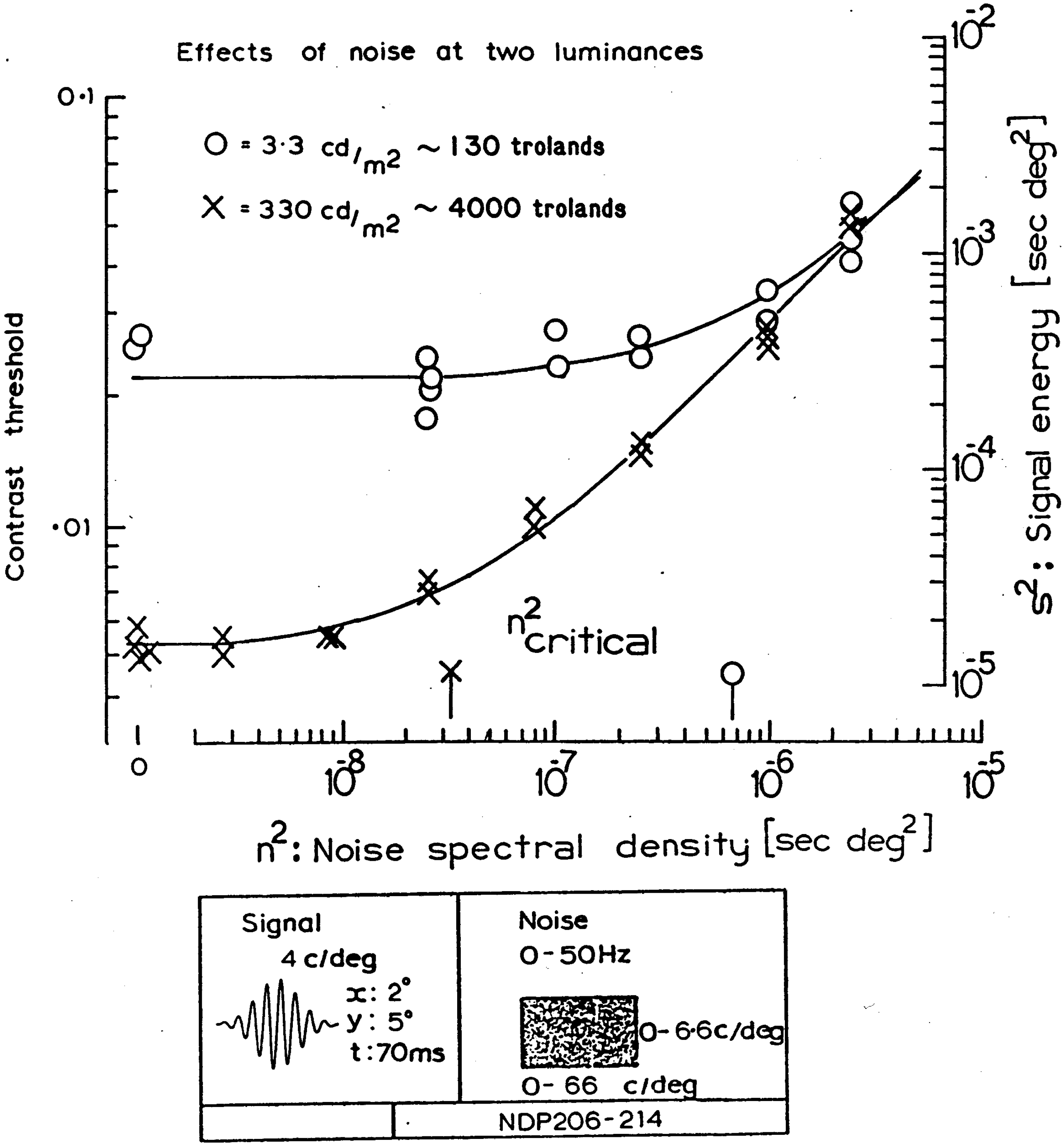
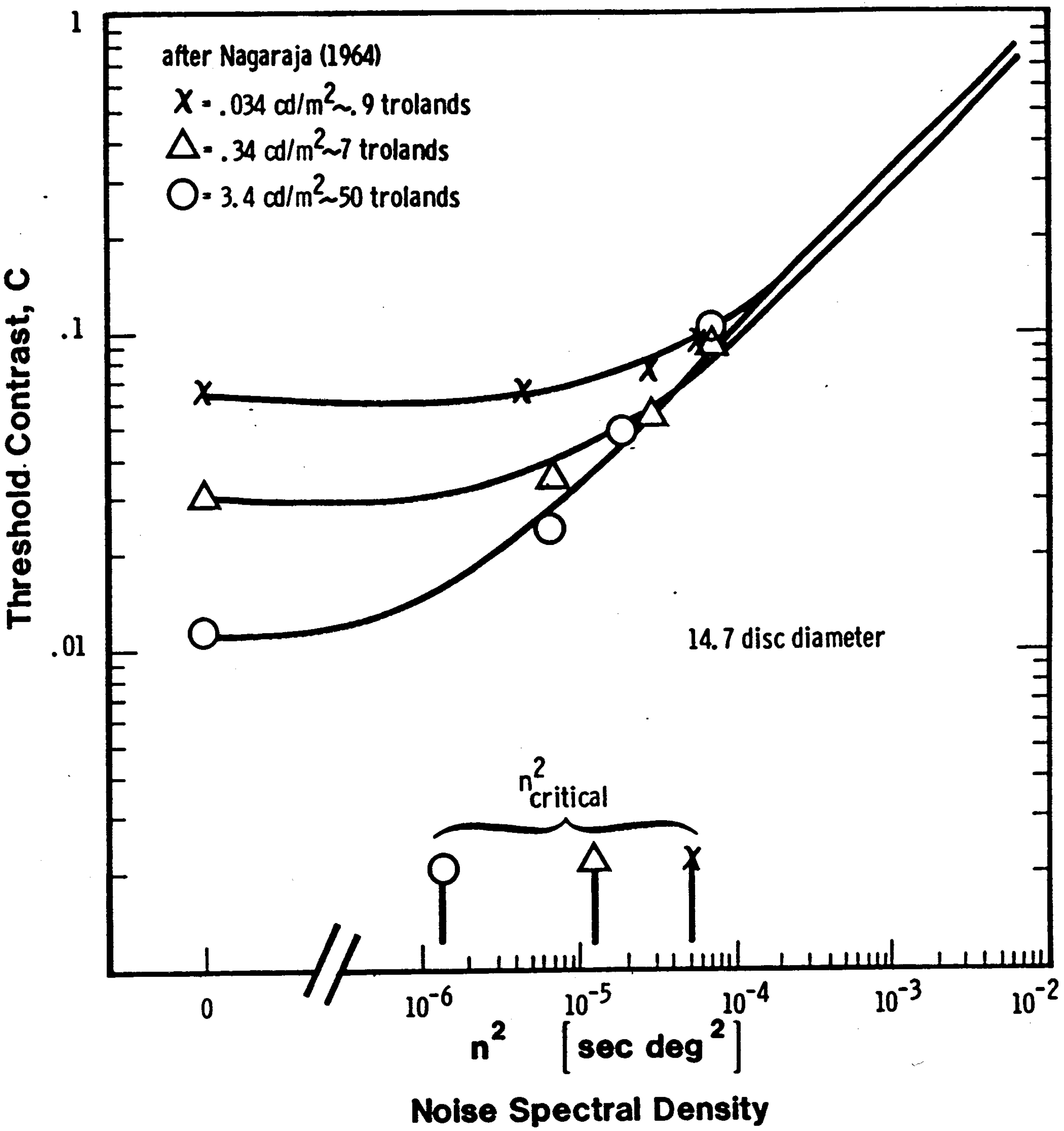

OLD AND NEW NAMES
Eye
Brain
S/n photons
Rose's "k" and "R"
Jones's "k"
d'
Rose's
"quantum efficiency of the eye"
Barlow's
"central efficiency"
Quantum efficiency
STIMULUS
RESPONSE
S/n photons
Transduction
S/n effective
Calculation
d'
Transduction efficiency
Calculation efficiency
Quantum efficiency

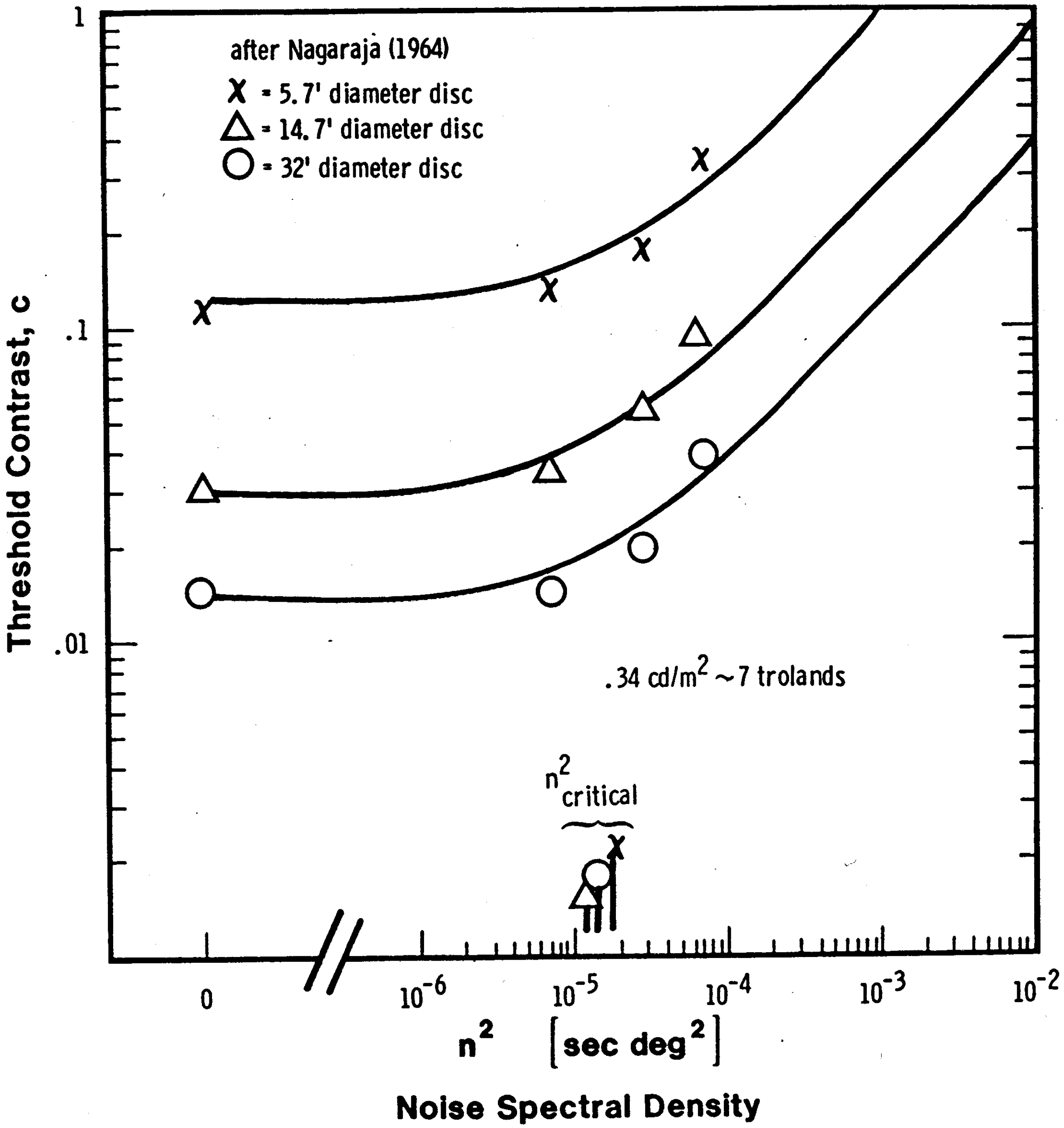
Figure 2.2

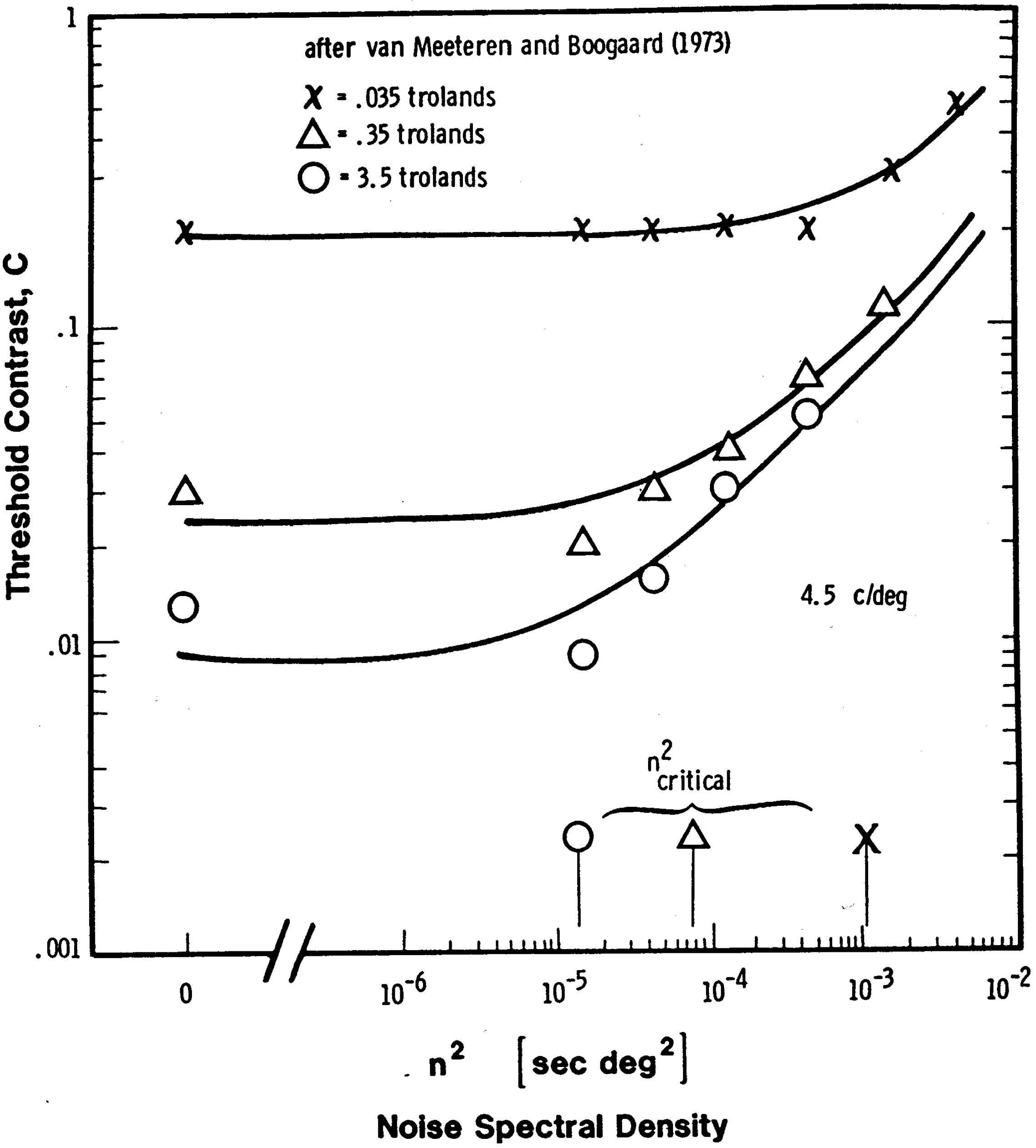
Interobserver Variability

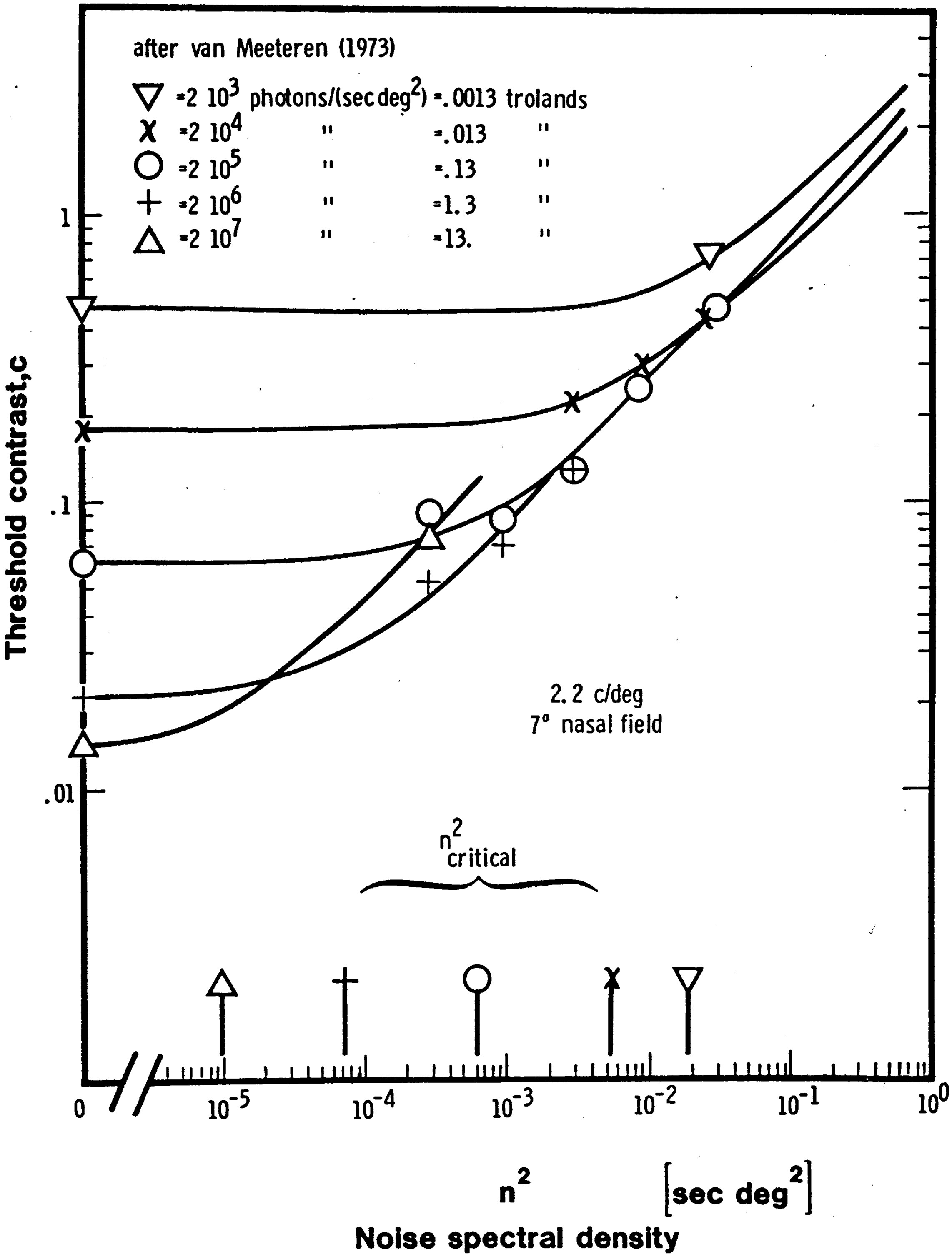



Effects of noise at two luminances
O = 3·3 cd/m2 ~ 130 trolands
X = 330 cd/m2 ~ 4000 trolands
Contrast threshold
0·1
·01
s^2: Signal energy [sec deg^2]
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$n^2_{critical}$
0
10^{-8}
10^{-7}
10^{-6}
10^{-5}
n^2: Noise spectral density [sec deg^2]
Signal
4 c/deg
x: 2°
y: 5°
t:70ms
Noise
0-50Hz
0-6·6c/deg
0- 66 c/deg
NDP206-214


after Nagaraja (1964)
X = .034 cd/m²~.9 trolands
△ = .34 cd/m²~7 trolands
○ = 3.4 cd/m²~50 trolands
Threshold Contrast, C
1
.1
.01
14.7 disc diameter
n²critical
0
10⁻⁶
10⁻⁵
10⁻⁴
10⁻³
10⁻²
n² [sec deg²]
Noise Spectral Density


after Nagaraja (1964)
X = 5.7' diameter disc
△ = 14.7' diameter disc
○ = 32' diameter disc
Threshold Contrast, c
1
.1
.01
.34 cd/m2 ~ 7 trolands
n2 critical
0
10-6
10-5
10-4
10-3
10-2
n2 [sec deg2]
Noise Spectral Density


after van Meeteren and Boogaard (1973)
X = .035 trolands
△ = .35 trolands
○ = 3.5 trolands
Threshold Contrast, C
1
.1
.01
.001
4.5 c/deg
n2 critical
0
10-6
10-5
10-4
10-3
10-2
n2 [sec deg2]
Noise Spectral Density


after van Meeteren (1973)
▽ =2 10^3 photons/(sec deg^2) =.0013 trolands
X =2 10^4 " =.013 "
O =2 10^5 " =.13 "
+ =2 10^6 " =1.3 "
△ =2 10^7 " =13. "
1
.1
.01
Threshold contrast,c
2.2 c/deg
7° nasal field
n^2 critical
0
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
n^2 [sec deg^2]
Noise spectral density

TRANSDUCTION EFFICIENCY

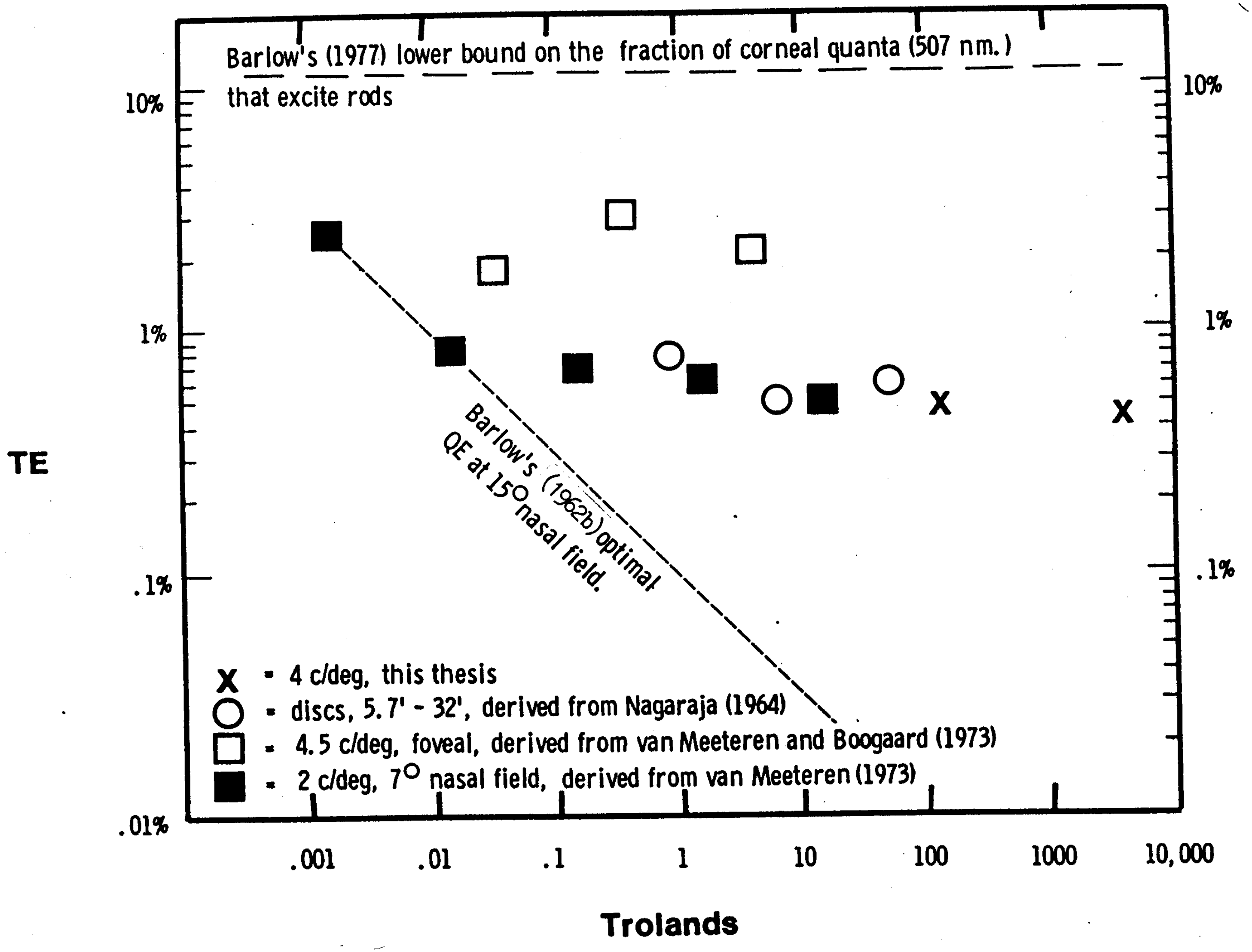


Figure 2.9

CHAPTER 3

THRESHOLDS IN NON-WHITE NOISE[1]

[1] Some of the spatial-frequency tuning results were reported at the 1980 meeting of the Association for Research in Vision and Ophthalmology in Orlando, Florida (Pelli, 1980). The data on temporal tuning were collected in collaboration with A.B. Watson.

ABSTRACT

When an observer is asked to detect a sinewave grating, is squared threshold proportional to the total noise power (including the equivalent noise) passed by a single linear filter ("channel") which depends on the signal, but not on the noise? Both hypotheses will be experimentally tested along several dimensions of spatio-temporal frequency. The hypotheses allow the modulation transfer function (MTF) of the filter to be derived from thresholds measured in bandlimited noise. By bandlimiting the noise in various dimensions the MTF may be determined over horizontal and vertical spatial frequencies and temporal frequency. In this way, the filter bandwidth (full bandwidth at half amplitude) is found to be about 7.5 c/deg horizontally (for a 5 c/deg horizontal grating), .6 c/deg vertically (for 1, 3, and 9 c/deg horizontal gratings), and 16 Hz temporally (for a .25 c/deg grating which was static, or temporally modulated at 4 or 16 Hz).

INTRODUCTION

Chapter 2 found that the squared contrast threshold for many patterns is proportional to the effective spectral density of a white noise background. This chapter will consider how different frequency components of the noise contribute to the threshold elevation of a sinusoidal grating. Chapter 2 was primarily concerned with effects of noise level (of white noise). This chapter examines effect of cut-off frequency and bandwidth (of bandlimited white noise[2]). The initial hypothesis will be that for any given test pattern there is a linear filter such that the observer's squared threshold is proportional to the total noise power passed by the filter, regardless of the noise spectrum. Later we will acknowledge that the observer may have many such linear filters available and need not always use the same filter to detect a given pattern.

The assumptions of Theory of Signal Detectability are not satisfied here, so detective efficiency is undefined. However the finding that threshold may be raised by noises whose spectra do not overlap the signal's spectrum is obviously relevant to understanding why the calculation efficiency, in white noise, can be so low. Furthermore, it is only by manipulation of noise bandwidth that one can determine at what bandwidth the noise becomes white, i.e. exceeds the bandwidth of the mechanism under study.

Several different lines of evidence suggest that human detection of spatial-frequency gratings is mediated by independent mechanisms, "channels",

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[2] "Bandlimited white noise" is not white noise. White noise has a constant spectral density and frequency components with independent amplitudes over the bandwidth of the mechanism under study. "Bandlimited white noise" is white over a stated band. This band usually will not include the entire bandwidth of the mechanism under study, and thus the noise will not be white.

each sensitive over one or two octaves of spatial frequency (on contrast adaptation see Blakemore and Campbell, 1969; on summation see Graham, 1977; on masking by gratings see Legge and Foley, 1980; noise masking will be reviewed below). We will see that effects of visual noise are consistent with this notion.

The approach taken here borrows much from audition. Fletcher (1940) found that the threshold for a tone increased with increasing bandwidth of a narrow band of noise (at constant spectral density) centered on the tone frequency, up to a critical bandwidth beyond which further increases in bandwidth had no effect. He reasoned that if our ear is composed of a bank of filters through which we can detect independently then only noise which goes through the same filter as the signal will have any effect on the detectability of the signal. The results of a replication of his experiment by Swets, Green and Tanner (1962) have been plotted on the right-hand side of Figure 3.1. The critical band is 130 Hz, a small fraction of the 1000 Hz signal frequency. Fletcher's notion of a critical band has been important in studies of the effects of visual noise too, as we will now see.

CRITICAL BANDS

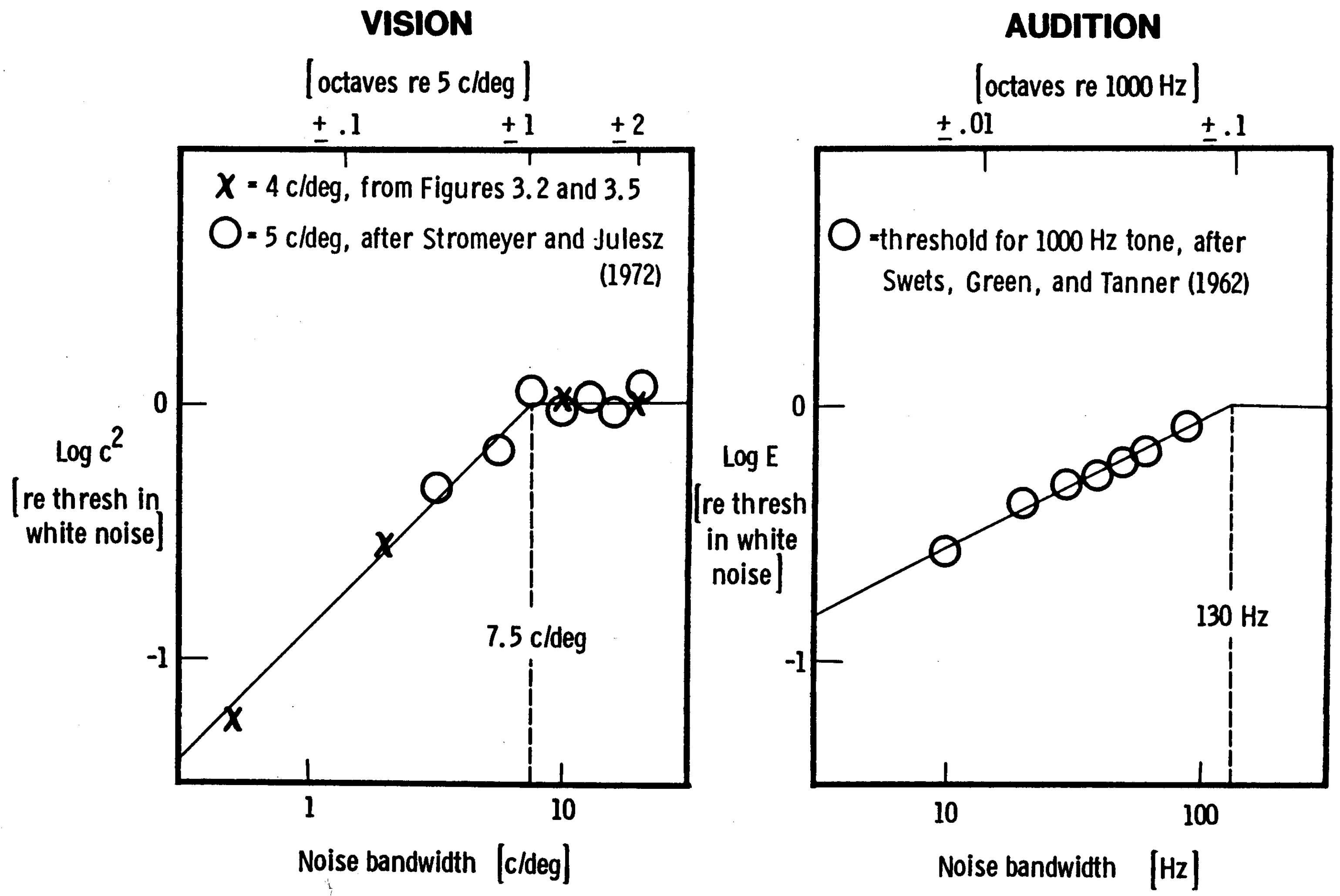


Figure 3.1

REVIEW

Visual thresholds in non-white noise

The review will be chronological.

Greis and Rohler (1970) used optical filtering techniques to produce a series of photographic transparencies of low-pass-filtered one-dimensional noise. By simultaneously projecting a sinusoidal grating and the noise they were able to determine 2IFC contrast thresholds (60% correct) for detection of the grating. They found that,

> "As long as the spectra of signal and background overlap or at least are in close proximity, the ratio of signal and interference [i.e. noise] energies[3] is decisive for subjective detectability. When the respective spectra are separated by a certain distance on the spatial frequency axis, however, they practically no longer affect each other; the detectability of the signal improves by a considerable amount."

Pollehn and Roehrig (1970) measured threshold contrast as a function of spatial frequency of the grating in the presence of noise backgrounds of several different spectra. Dynamic 2-dimensional white noise raised thresholds at all spatial frequencies, though more at middle frequencies, near 3 c/deg. Limiting the two-dimensional dynamic noise to a narrow band (bandwidth of 1.2 c/deg and center frequency 3.3, 10, or 16.7 c/deg) raised the grating thresholds most at the center frequency of the noise. Low-pass noise (0 to 1.2 c/deg) only slightly elevated threshold, maximally at 2 c/deg. The relative elevation curves had a bandwidth at half height of about two octaves.

Carter and Henning (1971) found that narrow-band noise was more effective

---

[3] By "energy" they meant contrast energy. Since the spectral density of the noise was nearly constant, constancy of their "ratio of signal and interference energies" implies proportionality of squared threshold contrast to noise bandwidth.

in raising the threshold of a 160 cycle (i.e. narrow spectrum) grating while broad band noise was more effective in raising the threshold of a single cycle (i.e. broad spectrum) grating. This too suggests size or spatial-frequency tuning of the detectors.

In the first thorough test of the notion of critical bands in vision, Stromeyer and Julesz (1972) measured grating thresholds in the presence of one-dimensional dynamic noise with low-pass, high-pass, and band-pass spectra. In their paper's final experiment Stromeyer and Julesz used a constant-spectral-density band of noise centered (on a logarithmic frequency scale) on the signal frequency. Their results have been replotted onto the lefthand side of Figure 3.1, along with similar data of mine. Their description is apt,

> "Masking [i.e. threshold] increased as the noise band was widened, up to a critical width of approximately ±1 octave, and did not increase further when the band was widened beyond this range. . . . The results demonstrate that a grating is masked only by noise whose spatial frequencies are similar to the grating frequency."

These results clearly distinguish our poor visual spatial-frequency selectivity from our acute auditory temporal-frequency selectivity: the auditory critical bandwidth is 13% of the tone frequency (or $\pm\frac{1}{10}$ octave); Stromeyer and Julesz's visual critical bandwidth (±1 octave) is about 150% of the grating frequency.

In 1973 Harmon and Julesz reported some now-famous observations on the visibility of block pictures of Abraham Lincoln. They started with a photograph, isotropically low-pass filtered (i.e. blurred) to leave only spatial frequencies up to 10 cycles per picture height. Coarse sampling (20 samples per picture height) did not reduce the information content, but yielded a "block picture" which most people find difficult to recognize as Lincoln.

The spectrum of this block picture differs from the original only in frequencies above 10 cycles per picture height. By selective filtering the authors showed that most of the masking effect was due to the sampling "noise" in the band (10 to 40 c/picture height) adjacent to the original spectrum (0 to 10 c/picture height), while little masking was caused by sampling "noise" more than two octaves away (i.e. above 40 c/picture height). Finally they tested effects of isotropic two-dimensional noise bandlimited to either 10 to 40 c/picture height or 40 to 70 c/picture height. As with the sampling "noise" the noise mask with the adjacent spectrum masked Lincoln much more effectively. No quantitative measurements of visibility were reported.

Diagnostic radiographs suffer from quantal noise due to the limitations on the x-ray dosage to which a patient may safely be exposed. This has led to several studies of the detectability of targets, usually discs, in static noise. Chapter 2 reviewed the groundbreaking work of Sturm and Morgan (1949).

The noise in conventional radiographs has a white spatial-frequency spectrum; its spectral density is constant up to very high spatial frequencies. Computed-tomographic (CT) scanners have recently been introduced for use in diagnostic radiology. It turns out that if projection data containing uncorrelated noise are used to perform a CT reconstruction, the noise in the reconstructed image possesses unusual correlations not found in ordinary radiographs. Hanson (1979) determined theoretically and empirically that the noise in the reconstructed CT image has a power spectral density proportional to spatial frequency. Hanson produced a CT image of a target of an array of discs arranged in columns of decreasing contrast, and rows of decreasing diameter. He also produced a similar image with white noise instead of CT noise. For each disc size the observer was asked to identify the last column (i.e. lowest contrast) in which he was "reasonably certain" he could see the disc. Thresholds in white noise were approximately inversely proportional to the disc diameter, in agreement with Rose (1948) and Sturm and Morgan (1949). Hanson found that, relative to the white noise, CT noise produced higher thresholds for small discs (diameters of 3' and 6') than for larger discs. These results are vaguely suggestive of spatial frequency or size-tuned masking effects, but firm conclusions are prevented by the discordance between the frequency manipulations of the noise and the size-specific, but broad-spectrum signals.

Chesters (1973) and Chesters and Hay (1979a, b) measured threshold contrasts (by a modified yes-no procedure) for light and dark discs on a static noise background. The background was a projection of photographic grain, and thus had a white noise spectrum, except when it was defocused. They found that defocusing the noise by 1 diopter lowered threshold only for discs with diameters less than 15' of arc. Chesters and Hay noted that an ideal disc detector would integrate light from the known signal area and ignore the rest of the picture. They constructed such a detector (a photomultiplier viewing through a signal-sized aperture) and determined the rms output as it was scanned over the noise. They showed that defocusing the noise by 1 diopter reduced the rms detector output only for expected signals with diameters less than 15' of arc, in agreement with the psychophysics. Effects for 2 diopters defocus were similar, with a critical size of 30': thresholds for discs smaller than 30' diameter fell as much as 8-fold, but there was a smaller two-fold fall in thresholds for smaller discs, which was not predicted. Their study provides evidence of size-specific detection.

Recently Henning, Hertz, and Hinton (1981) used static noise backgrounds (randomly chosen for each observation interval) and the 2IFC method to measure the effects of high and low-pass noise on the detection threshold of gratings of 1, 3, and 6 c/deg. Using Patterson and Henning's (1977) energy-detection-model analysis they derive filters centered on the signal frequencies with a bandwidth of approximately two octaves at half height.

They replotted Stromeyer and Julesz's (1972) results as log threshold versus log cut-off frequency of low- and high-pass noise and noted that the data were symmetric: threshold in low-pass rose as the 1.3 power of cut-off frequency, and threshold in high-pass noise fell as the -1.3 power of cut-off frequency.

CRITICAL BANDS

It may seem either obvious or mysterious to suppose that a frequency-selective device is sensitive only to the total noise power passed by a linear filter. Appendix 2 considers two possible models for a "channel": the cross-correlation detector and the energy detector. Both models are frequency-selective by virtue of a linear filter, and in both the squared threshold is proportional to spectral density of white noise. However the cross-correlation detector obeys our total-filtered-noise power hypothesis, while the energy detector does not. Following Green and Swets (1966) and Patterson and Henning (1977), the appendix shows that the squared threshold for a cross-correlation detector is proportional to a weighted integral of the noise spectrum,

$$c^2 \propto \int_{-\infty}^{+\infty} |H(f)|^2 \, n^2(f) \, df \qquad \text{(cross-correlation detector)},$$

while for the energy detector the fourth power of threshold is proportional to a weighted integral of the squared noise spectrum[4],

$$c^4 \propto \int_{-\infty}^{+\infty} |H(f)|^4 \, n^4(f) \, df \qquad \text{(energy detector)}.$$

If we consider only very-narrow-band white noise[5] so that $|H(f)|$ is approximately constant over the noise band then integration over that narrow

---

[4] The result for the cross-correlation detector is exact. The result for the energy detector neglects distortion products and is a good approximation only at high signal-to-noise ratios. Green and Swets (1966) give a good discussion of the plausibility of this assumption.

---

[5] That is,

$$n^2(f)=n^2$$

inside the band, and $n^2(f)=0$ outside.

band is equivalent to multiplication by the bandwidth of the band, W,

$$c^2 \propto W\ |H(f)|^2\ n^2 \qquad \text{(cross-correlation detector)}$$

$$c^4 \propto W\ |H(f)|^4\ n^4 \qquad \text{(energy detector),}$$

or, taking the square root,

$$c^2 \propto \sqrt{W}\ |H(f)|^2\ n^2 \qquad \text{(energy detector).}$$

Squared threshold in either detector is proportional to the noise spectral density, n^2, and to the power gain $|H(f)|^2$, i.e. squared MTF. However the two models depend differently on the noise bandwidth. The cross-correlation detector's squared threshold is proportional to W, the noise bandwidth, while the energy detector's squared threshold is proportional to $\sqrt{W}$, the square root of the noise bandwidth.

The two models are examples, showing that proportionality of squared threshold to noise level tells us nothing about dependence on bandwidth. The total-filtered-noise-power hypothesis predicts that squared threshold will be proportional to both bandwidth and spectral density of narrowband noise.

METHODS

Except for the differences noted below the methods of Chapter 2 apply. The main change was to bandlimit the noise to less than the bandwidth of the mechanism under study.

Signal

The signal was always a sinusoid with vertical bars, and Gaussian envelope. For some experiments the signal was also sinusoidally modulated in time.

For the first set of experiments there was no vertical attenuation. Examining the signal box of the inset in Figure 3.2 shows "x: 4°", indicating that the contrast fell to 1/e in 4° degrees horizontally, "t: 70ms", indicating that the contrast fell to 1/e in 70 ms temporally, and "y:[10°]", indicating that the grating was 10° high at constant contrast.

Noise

In the first experiments (Figures 3.1 to 3.11, except 3.3) the noise was dynamic one-dimensional noise. The "all-pass white noise" used in these experiments was white up to the Nyquist frequency, 23 c/deg, determined by the screen's 46 raster lines per degree. To produce low-pass or "high-pass" noise the "all-pass noise" was filtered by an appropriate 8-stage filter before the noise and signal were added and displayed.

Narrowband "heterodyne" noise with very steep skirts was produced by an amplitude-modulation technique apparently first used by Greenwood (1961). Appendix 4 presents the mathematics of this technique (and two others). An 8-stage low-pass filter with cut off at 1/4, 1/2, or 1 c/deg gave a narrow band of noise centered at 0 c/deg. This then modulated a carrier of the desired center frequency. The computer synthesized both the signal and the carrier for the noise. The resulting "heterodyne" noise has very steep skirts independent of center frequency.

Heterodyne noise is white on either side of the center frequency, but not taken as a whole, because the amplitude and phases of its frequency components are symmetric about the center frequency. Appendix 3 discusses the difference in effects of heterodyne and narrowband white noise on two hypothetical detection mechanisms. Appendix 4 explains the heterodyne technique, and two methods of producing narrowband white noise which future experiments may use to avoid the theoretical problems of heterodyne noise.

RESULTS

The left-hand side of Figure 3.1 plots squared threshold contrast (relative to threshold in white noise) versus noise bandwidth (at a constant noise level) on log-log scales. The "O"s are thresholds for 5 c/deg, replotted from Stromeyer and Julesz's (1972) Figure 10 (geometric mean of threshold contrast of both observers). The "X"s are threshold for 4 c/deg replotted from Figures 3.2 and 3.5. Each set of data has been normalized by the contrast threshold in white noise.[6] The straight lines, fitted by eye, show that squared threshold is proportional to noise bandwidth (as predicted by the total-filtered-noise-power hypothesis) up to 7.5 c/deg (±1 octave about 5 c/deg) and constant beyond that. Stromeyer and Julesz noted the width of the critical band, but not the log-log slope. Examination of Figure 3.1 shows that their data do not cover a sufficiently wide range of bandwidths to distinguish between slopes of 1/2 and 1 for squared threshold versus noise bandwidth. This result is consistent with cross-correlation detection and inconsistent with energy detection. The analogous auditory experiment is plotted on the right hand side of Figure 3.1

Figure 3.2 shows threshold for a 4 c/deg signal as a function of the spectral density of narrowband noise centered on 4 c/deg. The "X"s show results for a noise bandwidth of 2 c/deg. The triangles show results for a noise bandwidth of 1 c/deg. The smooth curves (fitted by eye) show that squared threshold is proportional to the sum of the critical and actual spectral densities. The critical spectral densities are 5.3×10^{-7} sec deg and 1.26×10^{-7} sec deg for noise bandwidths of 1/2 and 2 c/deg, respectively. This is unlike masking by gratings: there is only the barest hint of facilitation (near the knee of the curve the average threshold is about one

CRITICAL BANDS

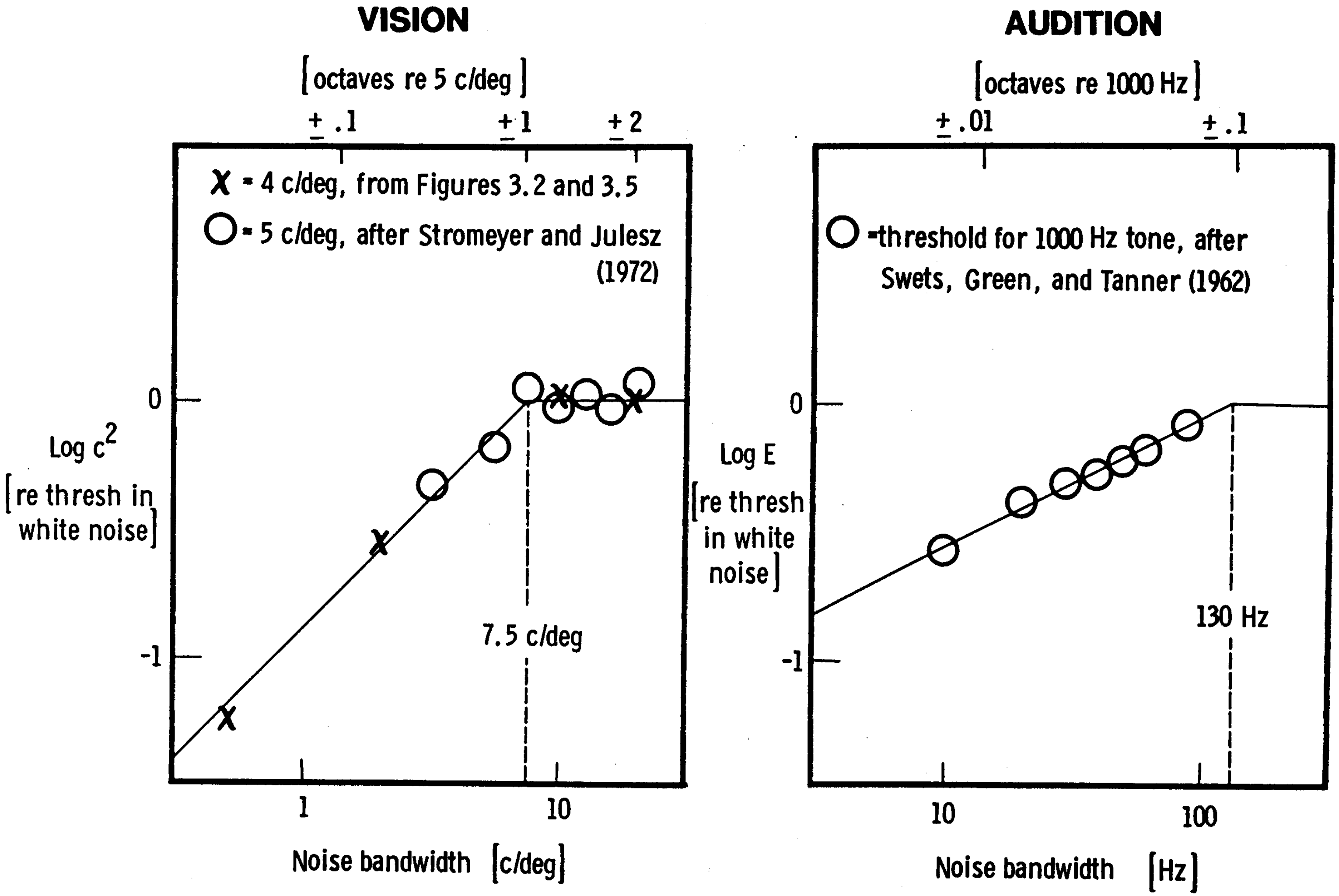


Figure 3.1

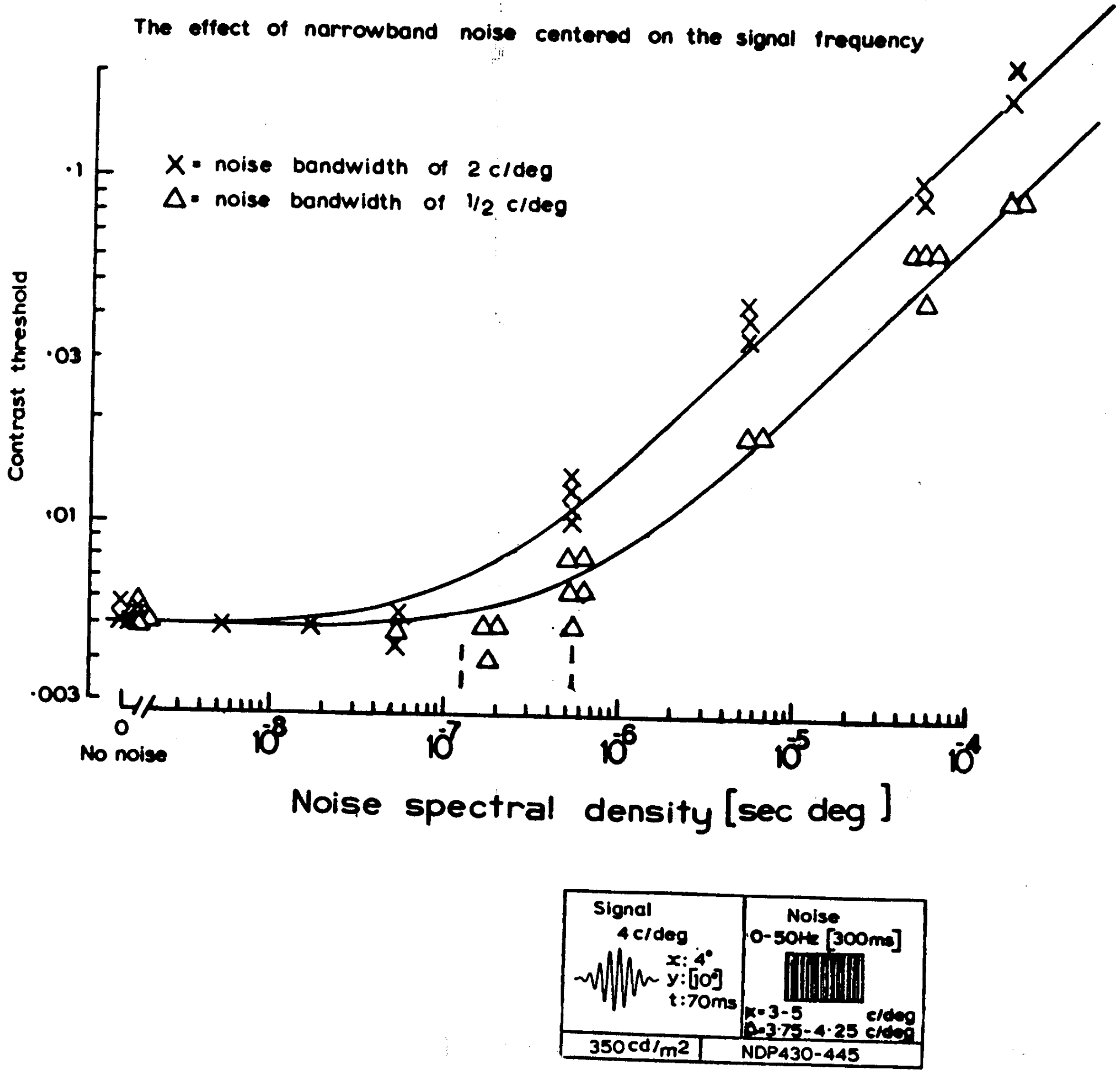

The effect of narrowband noise centered on the signal frequency
X = noise bandwidth of 2 c/deg
Δ = noise bandwidth of 1/2 c/deg
Contrast threshold
·1
·03
·01
·003
0
No noise
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
Noise spectral density [sec deg]
Signal
4 c/deg
x: 4°
y: [10°]
t: 70ms
Noise
0-50Hz [300ms]
x = 3-5 c/deg
Δ = 3·75-4·25 c/deg
350 cd/m2
NDP430-445

Figure 3.2

standard error, 1 dB, below the zero-noise threshold), and the log-log slope is 1, not .6. At high spectral densities squared threshold is proportional to noise level. This suggests repeating the following experiments at twice the noise level would only double the squared thresholds.

We have already seen that squared threshold in wideband noise is proportional to spectral density. Figure 3.2 plots threshold of a 4 c/deg signal as a function of the noise level of a narrow band of noise centered on

---

* There were several small differences between the experiments.

Stromeyer and Julesz used a noise level of n^2=6.9 10^{-6} sec deg, while my points are from a noise level of n^2=21 10^{-6} sec deg. It is unlikely that this difference in noise level matters, given the proportionality of squared threshold to noise level of both narrowband noise (see Figure 3.1) and broadband noise (see Chapter 2).

Stromeyer and Julesz's noise bands were symmetric about the signal frequency on a logarithmic scale (e.g. ±1 octave about 5 c/deg = 2.5 to 10 c/deg). My narrowband noise was symmetric on a linear frequency scale (i.e. 3.75 to 4.25 c/deg, and 3 to 5 c/deg, taken from Figure 3.2), and my broadband noise went down to zero frequency (i.e. 0 to 10 and 0 to 20 c/deg, taken from Figure 3.5).

In principle their method ("The subject adjusted the grating to threshold with a 1-dB step attenuator") is susceptible to criterion variations on the part of the observer. This seems particularly undesirable since a noise background that is always visible makes the observer's task more subtle than just determining whether or not the screen appears blank. There is evidence of this problem in their Figure 10 which shows the effect of noise bandwidth. As the noise bandwidth is increased most of the threshold curves oscillate up and down by more than 4 standard errors. It seems reasonable to expect that threshold should rise monotonically as the bandwidth of a noise mask is increased. They suggest that "the subject may adopt a different threshold criterion for judging gratings in narrowband vs. broad-band noise." My experiments avoided this problem by using 2AFC. Thresholds determined as a contrast required for a certain proportion correct responses on 2AFC are, at least in principle, immune to criterion variations (Green and Swets 1966). However the replot of Stromeyer and Julesz's data in Figure 3.1 (geometric mean of threshold contrasts from both their observers) shows that their results are very similar to mine in form though with somewhat more variability. It would seem that method- of adjustment can provide similar results to those found with a criterion-free method, but that the standard errors are misleading because they can be smaller than the variations due to systematic variations of criterion from condition to condition and from observer to observer.

the signal frequency. Two bandwidths were used, 1/2 and 2 c/deg. The curves (fit by eye) show that squared threshold is proportional to actual plus critical spectral density for these narrowband noises too. Quadrupling the bandwidth of the noise, from 1/2 to 2 c/deg, approximately quadrupled (the ratio is 4.2:1) the squared threshold for detection, as predicted by the total-filtered-noise-power hypothesis.

As noted in the Methods section, these experiments did not use narrowband Gaussian white noise (whose components are stochastically independent), instead they used heterodyne noise (which has a symmetric spectrum: the amplitudes and phases are symmetric about the center frequency of the band of noise). Appendix 3 evaluates the consequences of this on cross-correlation and energy detectors, and Appendix 4 shows how future experiments might create narrowband Gaussian white noise, and thus avoid the theoretical problems of heterodyne noise.

There is one point in common between the heterodyne and narrowband-white-noise masking data. When the noise band extended from 0 to 2 c/deg, Figure 3.2 (which used heterodyne noise) reports a threshold 9 dB higher than Figure 3.4 (which used narrowband white noise). Since the noise contrasts used differed by almost the same amount (ratio of 9.5 dB) the agreement is excellent. Thus, at least at this point, heterodyne noise had the same effects as narrowband white noise.

In the following sections various experiments will be presented that manipulate the noise spectrum in various dimensions. The particular choices of the type of noise were pragmatic; spectral manipulation of video signals is difficult. However each experiment varied the noise bandwidth in only one dimension, while the other spectral dimensions were kept fixed, and the total-filtered-noise-power hypothesis may be tested again and again in these different contexts.

CRITICAL BAND: ORTHOGONALLY

One of the important differences between vision and audition is the greater number of dimensions of visual stimuli. It is reasonable to ask what the channel's tuning is in the dimension orthogonal to the grating. The last section used one-dimensional dynamic noise, i.e. the vertical bandwidth was very small. This section will use two-dimensional dynamic noise. Various vertical bandwidths were used, but the horizontal bandwidth (33 c/deg) was sufficient to be called white. This was confirmed by finding identical results after increasing the bandwidth to 99 c/deg.

The two-dimensional spatial frequency of a horizontal 4 c/deg grating (i.e. with vertical bars) is

$$(f_x, f_y) = (4 \text{ c/deg}, 0 \text{ c/deg}).$$

We have seen that squared threshold was proportional to noise bandwidth in the horizontal dimension about the 4 c/deg test frequency. Here we ask if squared threshold is proportional to noise bandwidth vertically, about the 0 c/deg frequency of the test in that dimension.

Method

Three signal frequencies were used, 1, 3, and 9 c/deg, with circularly symmetric envelopes, falling to 1/e on a radius of 1.3° from the peak.

In this experiment the bandwidth of two-dimensional dynamic white noise mask was systematically reduced in the (vertical) dimension orthogonal to the (horizontal) frequency of the test gratings. The vertical bandwidth was controlled by adjusting the clock rate of the noise generator. The Nyquist frequency (half the sample rate) was taken as the nominal bandwidth of the noise. The horizontal bandwidth was independent of these clock rate manipulations.

The data in Figure 3.3 were collected at two viewing distances in order to ascertain whether the maximum attainable noise bandwidth (33 c/deg) at the normal viewing distance was effectively infinite. Tripling the bandwidth to 99 c/deg (by increasing the viewing distance[7]) did not significantly change thresholds so the results at both viewing distances have been combined.

Results

Figure 3.3 plots contrast threshold versus vertical bandwidth. As the fitted lines show, squared threshold is proportional to noise bandwidth, up to a critical bandwidth of about .6 c/deg, and constant beyond that. The fit to the 1 and 9 c/deg results is good; the fit to the 3 c/deg results is only fair.

The critical bandwidths are remarkably narrow, and imply linear integration, vertically, over at least a degree. This suggests the channels were well matched to the 2.3° vertical extent (at half-height) of the grating. A cross-correlation by the (known) signal would have a critical bandwidth of .27 c/deg,[8] which is about half that of the human observer.

It is surprising that the critical bandwidth should be the same for test frequencies differing by nearly an order of magnitude, but this result may depend on using, as here, the same vertical extent for all spatial frequencies.

---

[7] In case the reader misunderstand, I hasten to add that changing the viewing distance involved more than just taking a few steps backward. It required increasing the power of the noise in order to keep the same spectral density, increasing the physical scale of the signals in order to keep the same spatial frequency and extent (from the observer's point of view), and reducing the cut-off frequency of the low-pass filter in order to keep the same vertical bandwidth (from the observer's point of view).

---

[8] A vertical weighting function of $e^{-(y/1.3°)^2}$ has a Fourier transform,

$$1.3° \sqrt{\pi}\, e^{-(f_y\, 1.3° \sqrt{\pi})^2}.$$

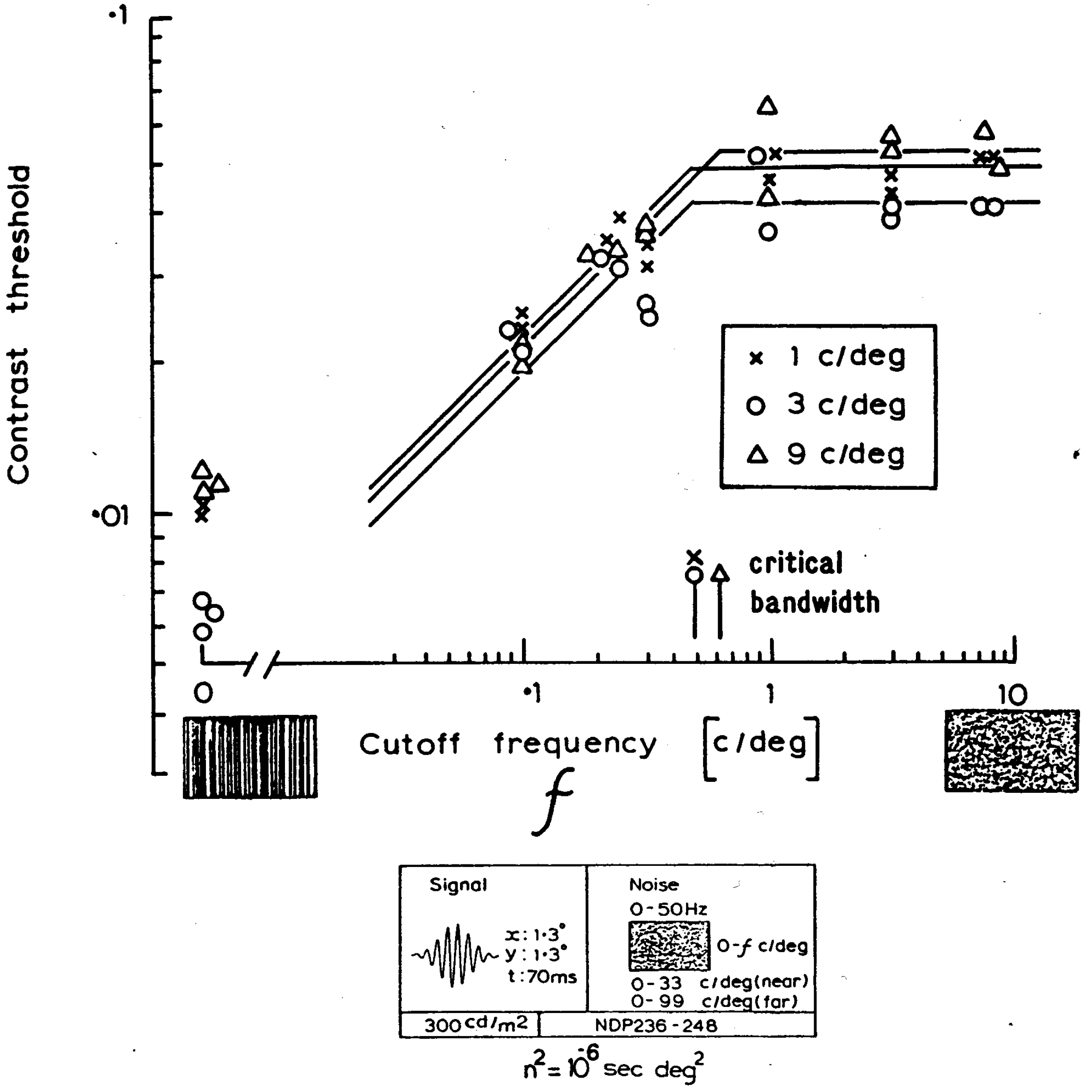
The critical bandwidth (single-sided) is

$$\int_0^\infty [e^{-(f_y\, 1.3° \sqrt{\pi})^2}]^2\, df_y$$

$$= 1/(1.3°\ 2 \sqrt{2})$$

$$= .27 \text{ c/deg}.$$

Effect of the orthogonal bandwidth of the noise.



Assuming that the each channel's tuning is separable into horizontal and vertical spatial frequencies, the .6 c/deg critical bandwidth vertically implies orientation tunings of ±17° at 1 c/deg, ±6° at 3 c/deg, and ±2° at 9 c/deg. Campbell and Kulikowski (1966) determined the orientation tuning for masking by narrowband one-dimensional dynamic noise. Threshold for a 10 c/deg vertical grating as a function of mask orientation had a bandwidth of ±12° at half height. Their ±12° tuning (at 10 c/deg) is much broader than the ±2° tuning (at 9 c/deg) deduced here on the assumption of separability. Clearly the two-dimensional spatial frequency tuning for detection of a grating needs more research.

SPATIAL-FREQUENCY TUNING

The review of the vision literature on thresholds in non-white noise showed that detection seems to be mediated by frequency-selective mechanisms, "channels". However no one has yet collected data on thresholds in noise which would distinguish various models of a frequency-selective channel. We have seen that squared threshold is proportional to the critical plus actual spectral densities of white noise (Chapter 2), and narrowband noise (Figure 3.2).

The hypothesis that squared threshold is proportional to total noise power at the filter output implies that the squared threshold in the presence of a sum of two noises with non-overlapping spectra should equal the sum of the squared thresholds in the presence of each noise alone.

This will be examined in several ways. Figure 3.2 will compare thresholds in 1 and 2 c/deg bandwidth narrowband noise, but the crucial test will come from measurements of threshold in low-pass, high-pass, and narrowband noise. All the experiments will use high noise levels so that threshold is raised many-fold over its noise-free value, and thus the equivalent noise may be neglected.

The experimental test rests on the fact that low-pass noise or high-pass white noise may be described as a sum of narrowband noises occupying contiguous, non-overlapping bands. The question is whether the squared thresholds in low- or high-pass noise may be predicted by the sum of the squared thresholds in the component narrowband noises. Or, conversely, is the squared threshold in narrowband noise from $f-\Delta$ to f predicted by the difference of the squared thresholds in low-pass (or high-pass) noise with cut-off frequencies of f-Δ and f c/deg?

If the hypothesis succeeds then we may conclude that squared threshold is proportional to the noise power passed by a linear filter whose squared modulation transfer function is given by the plot of squared threshold versus center frequency of narrowband noise.

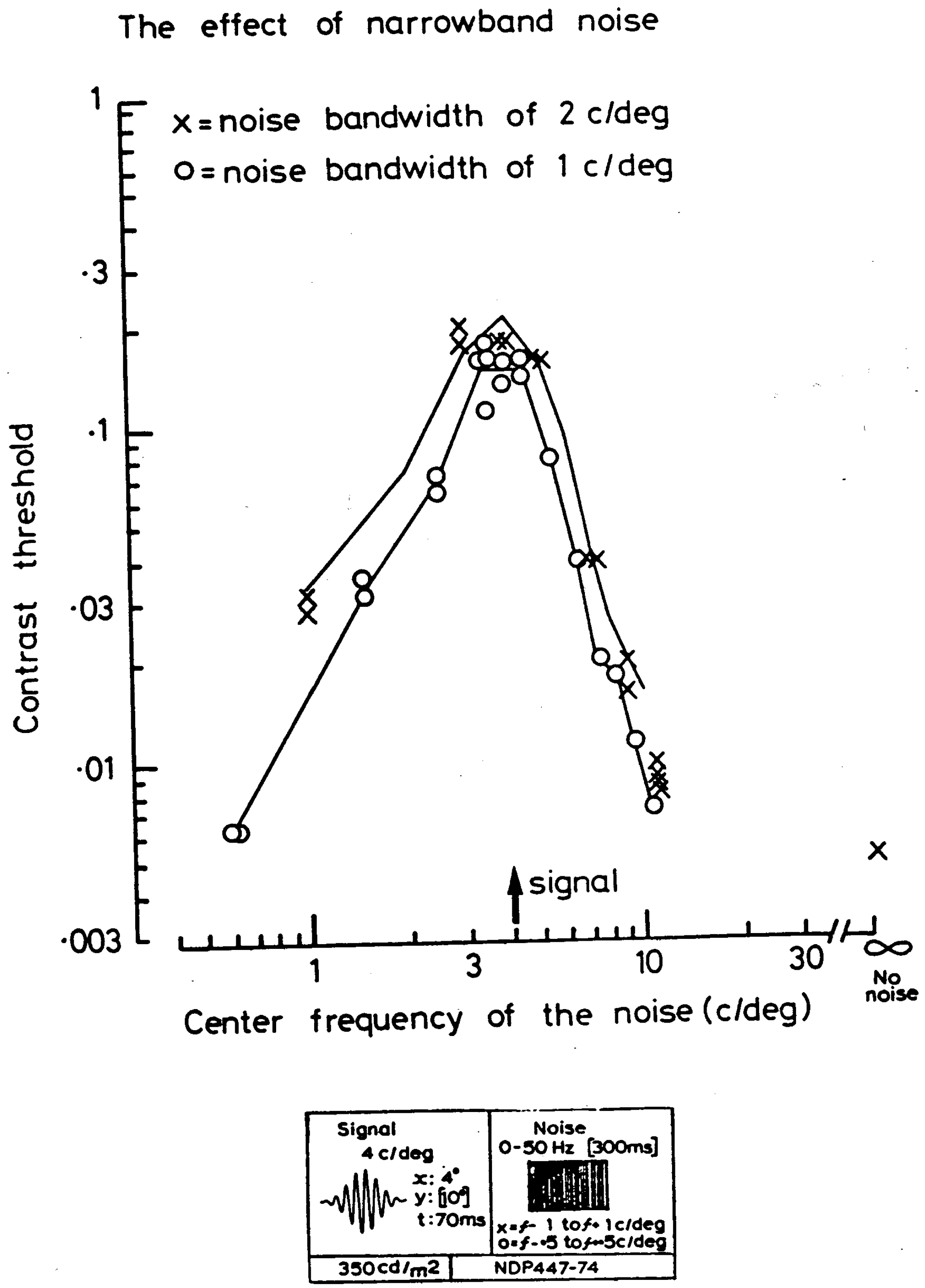
Published data on thresholds in noise are not suitable for a thorough test of the linear filter hypothesis. Stromeyer and Julesz (1972) and Henning, Hertz, and Hinton (1981) reported threshold in high- and low-pass noise. For our test we would need measurements with high- and low-pass noise at each cut-off frequency, but they used only low-pass noises with cut-off frequencies below the signal frequency, and only high-pass noises with cut-off above the signal frequency. They do report high- and low-pass thresholds with cut-off at the signal frequency, and according to our hypothesis the sum of the squared thresholds in high and low-pass noise with the same cut-off frequency should equal the squared threshold in all-pass noise. Unfortunately Stromeyer and Julesz did not report the three measurements in a compatible way. For all-pass noise the absolute threshold is reported, while for the low and high-pass noise, thresholds were reported only as elevations relative to threshold in the absence of noise, which was not reported. The Henning et al. result will be discussed along with mine.

Methods

The methods were exactly as described for Figure 3.2.

Results

Figure 3.4 plots (as "O") threshold of a 4 c/deg grating as a function of the center frequency of a band of noise 1 c/deg wide. Interpreted as a channel


The effect of narrowband noise
x = noise bandwidth of 2 c/deg
o = noise bandwidth of 1 c/deg
Contrast threshold
1
·3
·1
·03
·01
·003
signal
1
3
10
30
∞
No noise
Center frequency of the noise (c/deg)
Signal
4 c/deg
x: 4°
y: [10°]
t: 70ms
Noise
0-50 Hz [300ms]
x=f-1 to f+1 c/deg
o=f-·5 to f+·5 c/deg
350 cd/m2
NDP447-74

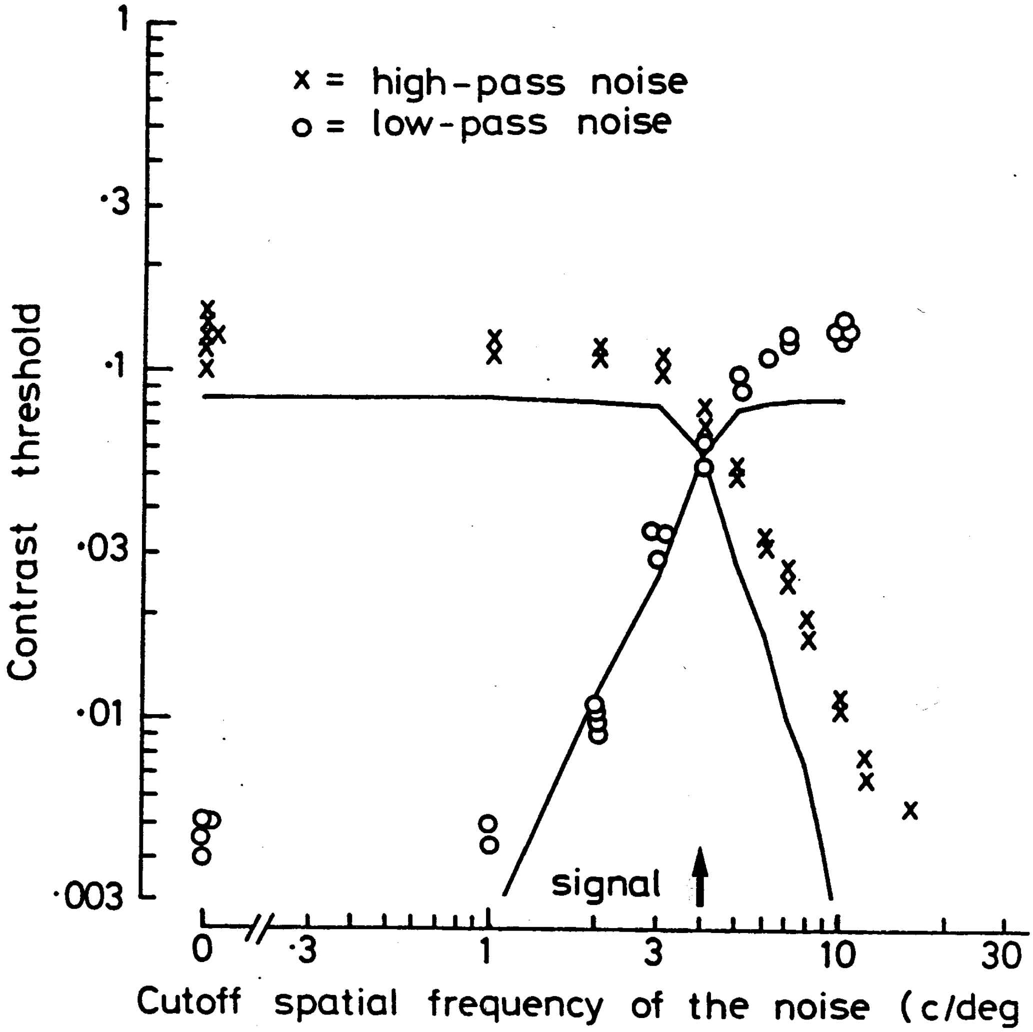
shape, the function has a full width at half maximum of one octave (i.e. 3 c/deg to 6 c/deg). Results for a 2 c/deg noise bandwidth are also plotted (as "X"). The solid curve through these results is a prediction from the narrower bandwidth results. Squared threshold in each 2 c/deg band of noise is predicted by the sum of squared thresholds in the 1 c/deg component bands of noise.

Figure 3.5 plots threshold as a function of cut-off frequency of high-pass ("X") or low-pass ("O") noise. The continuous curves are predictions from the narrowband noise data (for details see below: "off-frequency seeing"). The set of measurements was made three times, but only the last set is shown. Each repetition found lower thresholds for those conditions in which the cut-off frequency of the noise was near the signal frequency. The final repetition gave the results shown. The fall in threshold from the second to the third set of measurements was small, suggesting that an asymptote had been reached. Figure 3.9 documents this practice effect, which will be discussed further below ("Off-frequency looking").

The next experiments compare results with several test frequencies. Figure 3.6 plots the threshold of 4 c/deg (replotted from Figure 3.4) and 2.5 c/deg (plotted as "O") signals as a function of the center frequency of 1 c/deg bandwidth narrowband noise. A curve connecting all the 4 c/deg results, after shifting to the left by 2.5:4, provides a passable fit to the 2.5 c/deg results.

In Figure 3.7, thresholds in low-pass noise are plotted for signal frequencies of 2.5 ("Δ"), 3 ("X"), and 4 c/deg ("O", replotted from Figure 3.5). The curves have very similar shapes, differing principally in having slightly different asymptotes in all-pass noise. If the three signal frequencies had been detected by the same channel then the curves would have been vertical translations of one another, reflecting the channel's different

The effects of high-pass and low-pass noise.



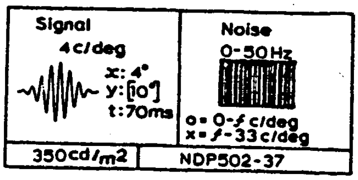


Figure 3.6

The effect of narrowband noise on two different signal frequencies.

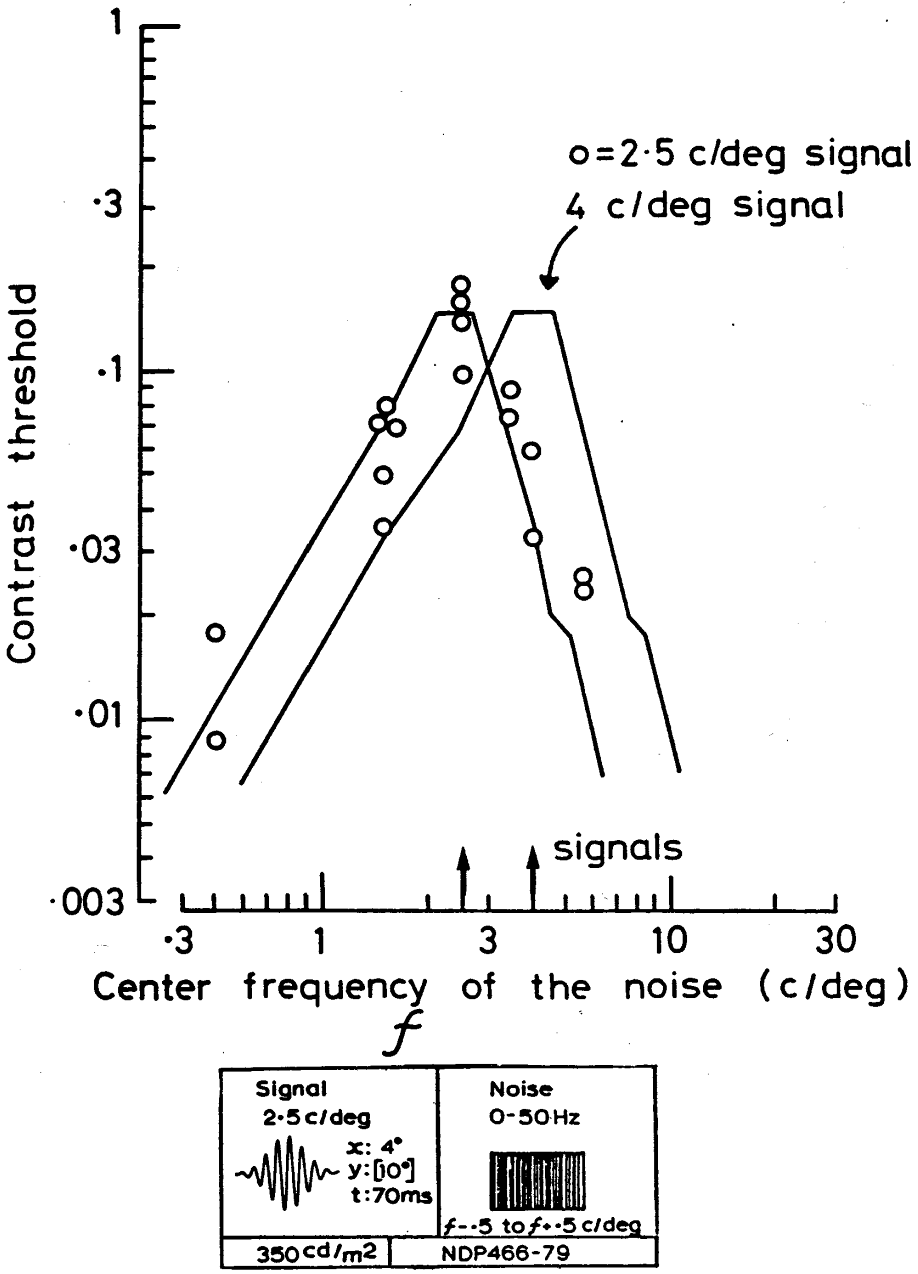
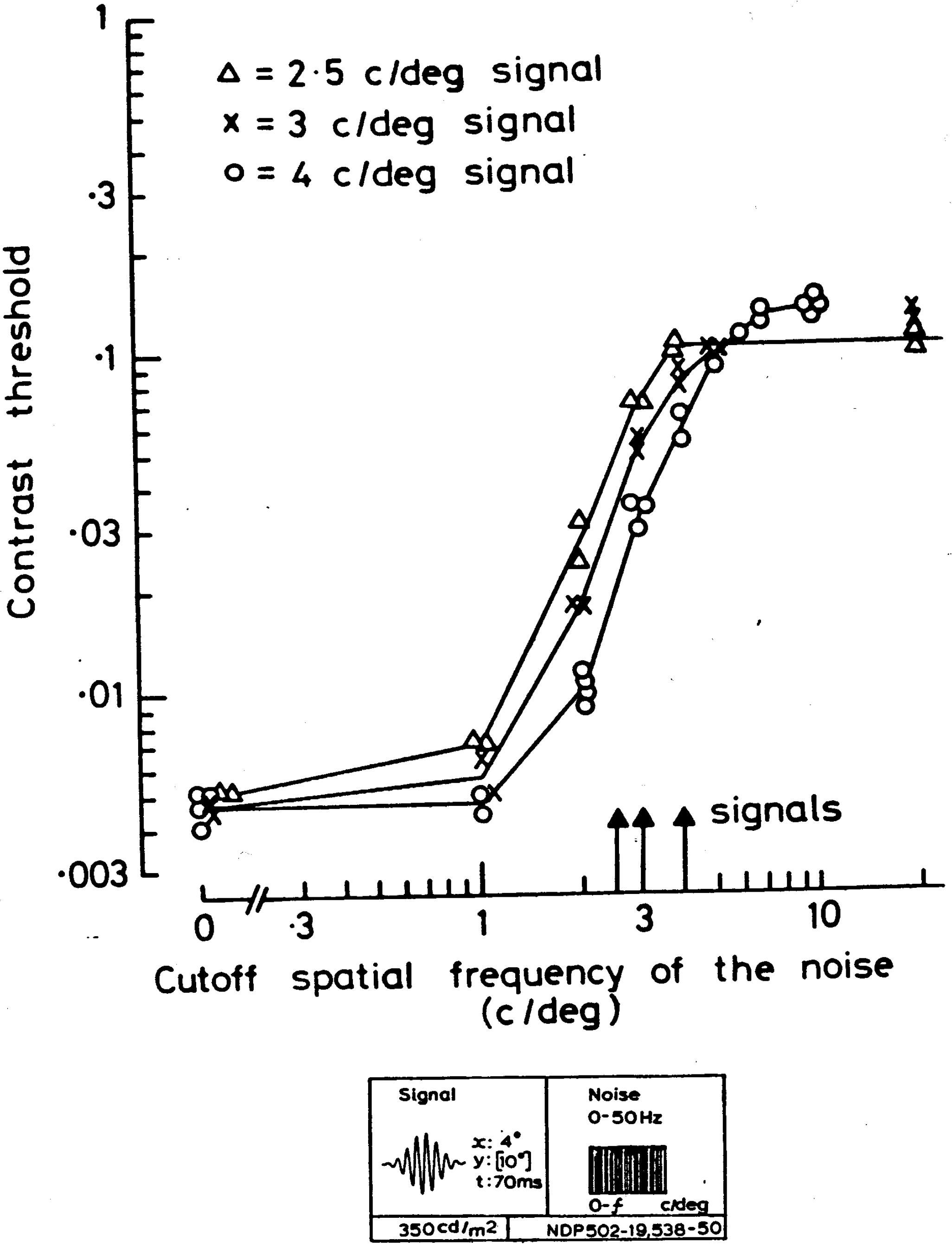


Figure 3.7

The effect of low-pass noise on three signal frequencies.



sensitivities to the three signal frequencies. Instead the curves translated with the signals along the frequency axis suggesting that near 3 c/deg there must be at least one channel every .5 c/deg. Thus near 3 c/deg the channel density is at least 4 channels per octave.

According to the total-filtered-noise-power hypothesis threshold in low- or high-pass noise should be sums of threshold in narrowband noise. The continuous lines in Figure 3.5 are predictions of the low-pass and high-pass results from the narrowband results of Figures 3.4. The predictions are much too low, particularly for all-pass noise.

A similarly unsuccessful prediction may be made using just the low- and high-pass data. Low- and high-pass noise with the same cut-off frequency are complementary parts of all-pass noise. Figure 3.8 shows predictions of the squared all-pass threshold by the sum of complementary squared low- and high-pass thresholds as a function of the cut-off frequency. The prediction is too low by about 2 dB near the signal frequency.

The next section will interpret these discrepancies as evidence that the observer used different channels when the noise was primarily above or below the 4 c/deg test frequency.

In this section we have tested two aspects of the total-filtered-noise-power hypothesis. In order to highlight the importance of these tests, two simple detectors were considered as examples. The hypothesis predicts that squared threshold should be proportional to noise level (true of both cross-correlation and energy detection), which was confirmed, and bandwidth of narrowband noise (true of cross-correlator, but not of energy detector) which was also confirmed. Furthermore the hypothesis predicts a very

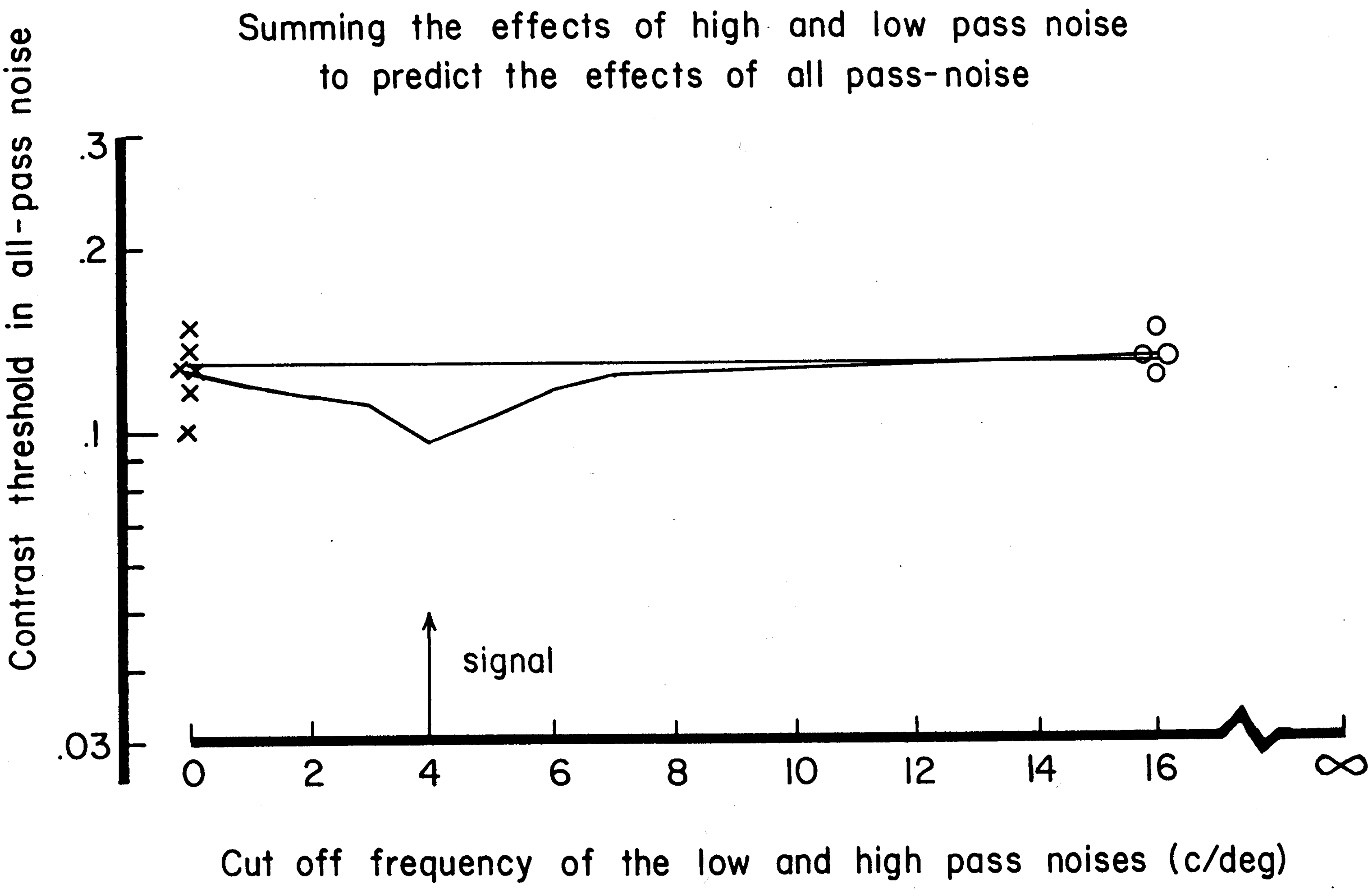

Summing the effects of high and low pass noise
to predict the effects of all pass-noise
Contrast threshold in all-pass noise
.3
.2
.1
.03
signal
0
2
4
6
8
10
12
14
16
∞
Cut off frequency of the low and high pass noises (c/deg)

Figure 3.8

general additivity: the sum of squared thresholds in several independent noises should equal the squared threshold in the sum of the noises[9].

Attempts to predict the effects of low and high-pass noise from those of narrow-band noise met with failure. Similarly, thresholds in low and high-pass noise with the same cut-off frequency underestimated the all-pass threshold when the cut-off frequency was near the signal frequency. The assumption that a signal is always detected by the same channel will now be dropped.

Off-frequency looking

Fletcher's critical band technique is not well suited for determining the MTF of the channel filter since the channel filter may be asymmetric. Patterson (1974) measured auditory thresholds for a tone as a function of its distance in frequency from the edge of a broad band of noise with very sharp skirts. Patterson's analysis assumed that a given signal is always detected by the channel with maximum gain at the signal frequency. It was subsequently realized that in the presence of noise at only one side of the signal frequency the best signal-to-noise ratio would in general be obtained by a filter whose maximum sensitivity was near but not at the signal frequency. Patterson and Nimmo-Smith (1980) found that their observers could listen "off-frequency" to lower their thresholds by as much as 5 dB.

An analogous off-frequency looking hypothesis is given partial support by the evidence that off-frequency looking seems to require practice. One might

---

[9] This assumes each noise has sufficient power to raise threshold well above the zero-noise level. More generally, the prediction is that the sum of squared thresholds (each minus squared zero-noise threshold) in several independent noises should equal the squared threshold (minus squared zero-noise threshold) in the sum of the noises.

expect that when first presented with a detection task an observer would use a channel centered at the signal frequency, and would only gradually learn to look off-frequency as he notices correlation between appearance and the subsequent feedback. Figure 3.8 shows prediction of the (squared) all-pass threshold by the sum of squared low- and high-pass thresholds. There is a discrepancy when the cut-off is near the signal frequency. Henning, Hertz, and Hinton (1981) found this too.

A complete set of thresholds in low- and high-pass noise were measured three times. Figure 3.9 shows the effect of practice for three conditions of interest: all-pass noise, and low- and high-pass noise with cut off at the signal frequency (4 c/deg).

As reported in Chapter 2, threshold in all-pass noise showed no systematic change with practice. Grouped by days, the second day thresholds in all-pass noise were slightly higher that each of the other days (T-test significant at the .1 level), but all-pass thresholds were essentially the same on the first, third, and fourth days (T-test not significant at the .1 level).

Improvement with practice is evident in both low- and high-pass noise conditions. Grouped by days the thresholds in high-pass noise fell about 40% (5 dB) and the threshold in low-pass noise fell about 30% (3 dB) from the first to the last day. A T-test showed that both differences are significant at the .025 level.

The channel shape derived by Patterson and Nimmo-Smith was in good agreement with that found by Patterson (1976) using notch noise (i.e. white noise with a narrow frequency band removed), a technique designed to minimize off-frequency listening, but was twice as broad as that derived by earlier

The effect of practice on thresholds in noise

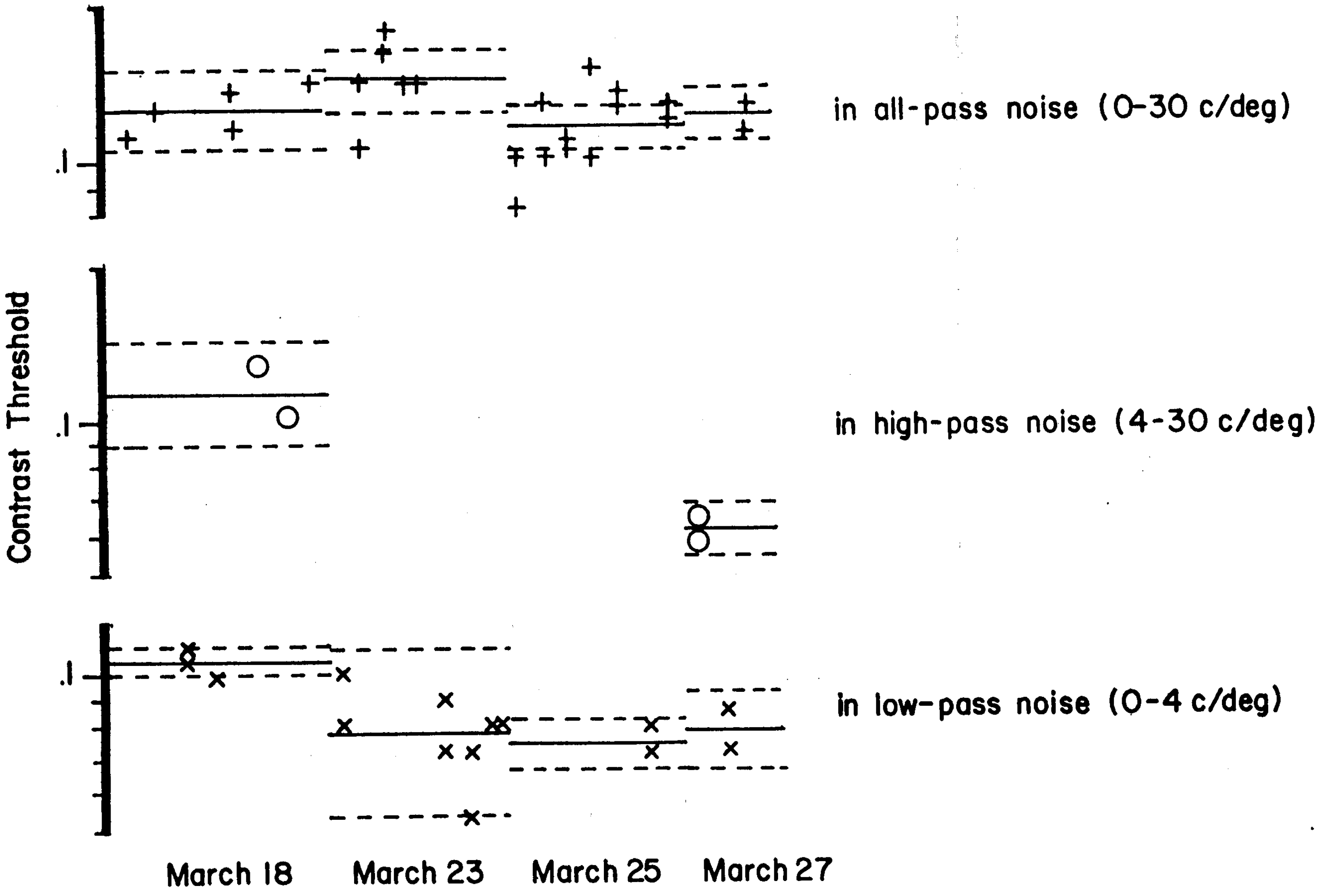


Figure 3.9

studies using narrow-band, or low- or high-pass noise (e.g. Patterson 1974). The notch-noise technique is elegant, but requires assuming that the channel is symmetric on a linear frequency scale, which is nearly true in audition, but is manifestly false for spatial-frequency channels in vision.

Assume that threshold is always determined by the filter with the best signal-to-noise ratio, s/n . What filter shape would give rise to the data of Figures 3.4 and 3.5? Patterson and Nimmo-Smith (1980) attempted to answer a similar question for their hearing data but found that the filter shape "solution" was unstable, that is, insignificant changes in the data produced violent changes in the solution. They overcame the difficulty by constraining the solution. They assumed the channel shape could be described by a "rounded exponential", $(a+bx+cx^2+dx^3)e^{-k|x|}$. This function has the parameters a,b,c,d, and k, to adjust the shape in various ways, but constrains the solution to conform to their prior belief that the channel has a roughly Gaussian shape.

One would like the experiment to dictate the model. Allowing the data only to adjust a few parameters of an assumed model fails to satisfy this desire. However by the total-filtered-noise-power hypothesis squared threshold is a weighted integral of the filter's squared MTF over the noise band. Small wiggles in the filter shape would not affect the results of Figure 3.5. Indeed Patterson and Nimmo-Smith's solution contained non-monotonic wiggles on both skirts of the filter which they dismissed as artifacts of the "rounded exponential". The wiggles may indicate that their constraints were only just sufficient to obtain a stable solution.

I have less data than they did. Therefore I must constrain the solution more narrowly. We will assume the MTF has the form,

$$|H(f)| = \begin{cases} (f/f_l)^l & \text{if } f<f_l \\ 1 & \text{if } f_l<f<f_h \\ (f/f_h)^{-h} & \text{if } f_h<f \end{cases}$$

Appendix 5 shows that if the channels all have this "flat-top power law" shape then (1) one channel should account for all the low-pass noise thresholds and the threshold in narrowband noise below the signal frequency, and (2) a second channel should account for all the high-pass noise thresholds and the thresholds in narrowband noise above the signal frequency.

Figure 3.10 shows fits of this flat-top power law model to the low- and high-pass data (assuming the total-filtered-noise-power hypothesis applies to each channel). Initial fits yielded similar slopes for the skirts so the channel was constrained to be symmetric (i.e. $l=h$ in the above equation). The center frequency was varied independently for the low and high-pass data. The best fitting bandwidth was 1.5 octaves (at half height) for the low-pass data, and 2 octaves for the high-pass data; both fits are shown.

The fits were made by eye. This was straightforward because the sloping parts of the data determine the steepness of the skirts, so for each bandwidth that I selected, a simple computer program determined the center frequency which would correctly predict the all-pass threshold.

Figure 3.11 shows prediction of the narrowband thresholds from the best fit to the low- and high-pass thresholds. As discussed above the expectation is that one channel describes the high-pass and narrow-band-above-signal-frequency results, and that another channel describes the low-pass and narrow-band-below-signal-frequency results. This expectation is largely born out; the largest discrepancy is that the

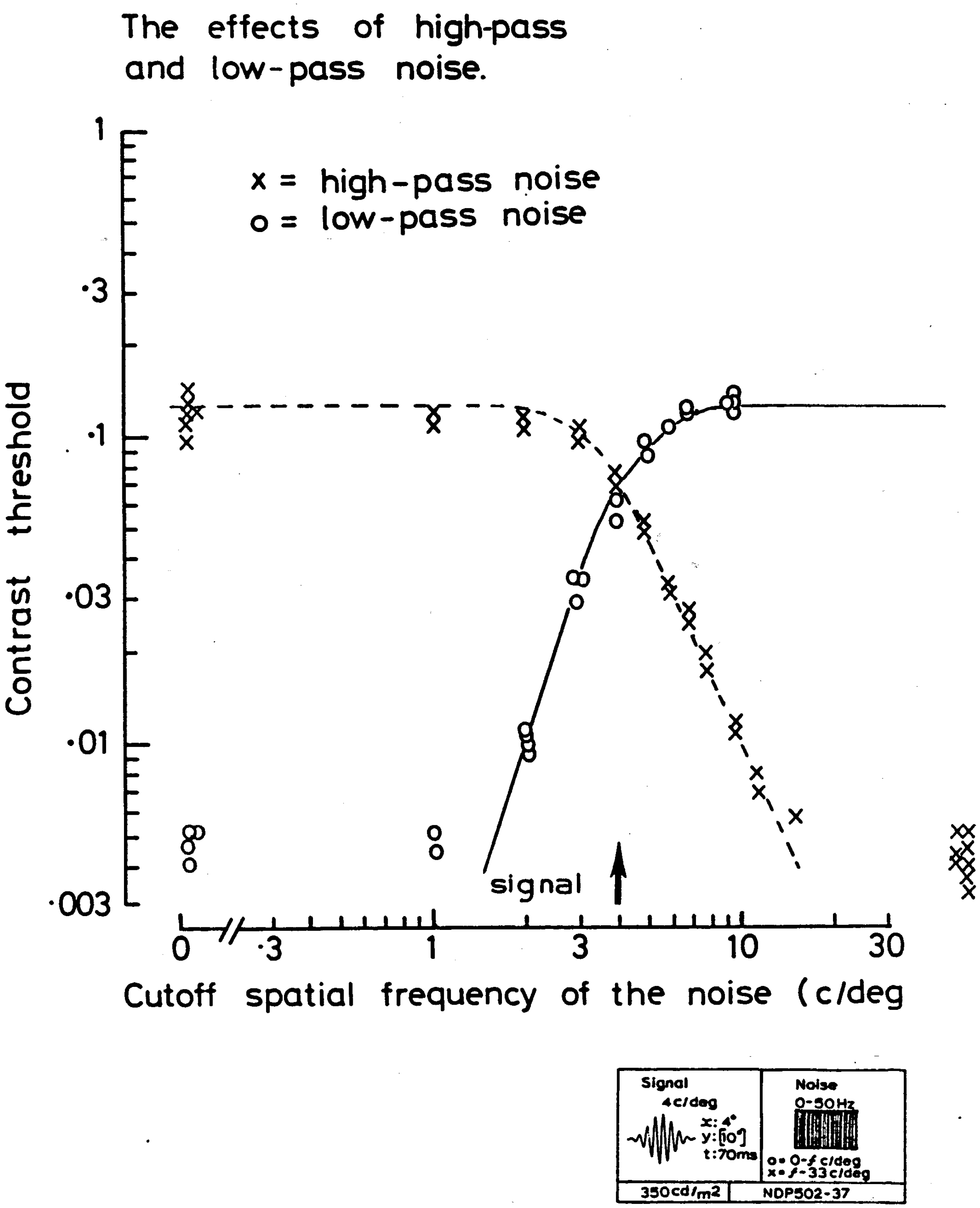
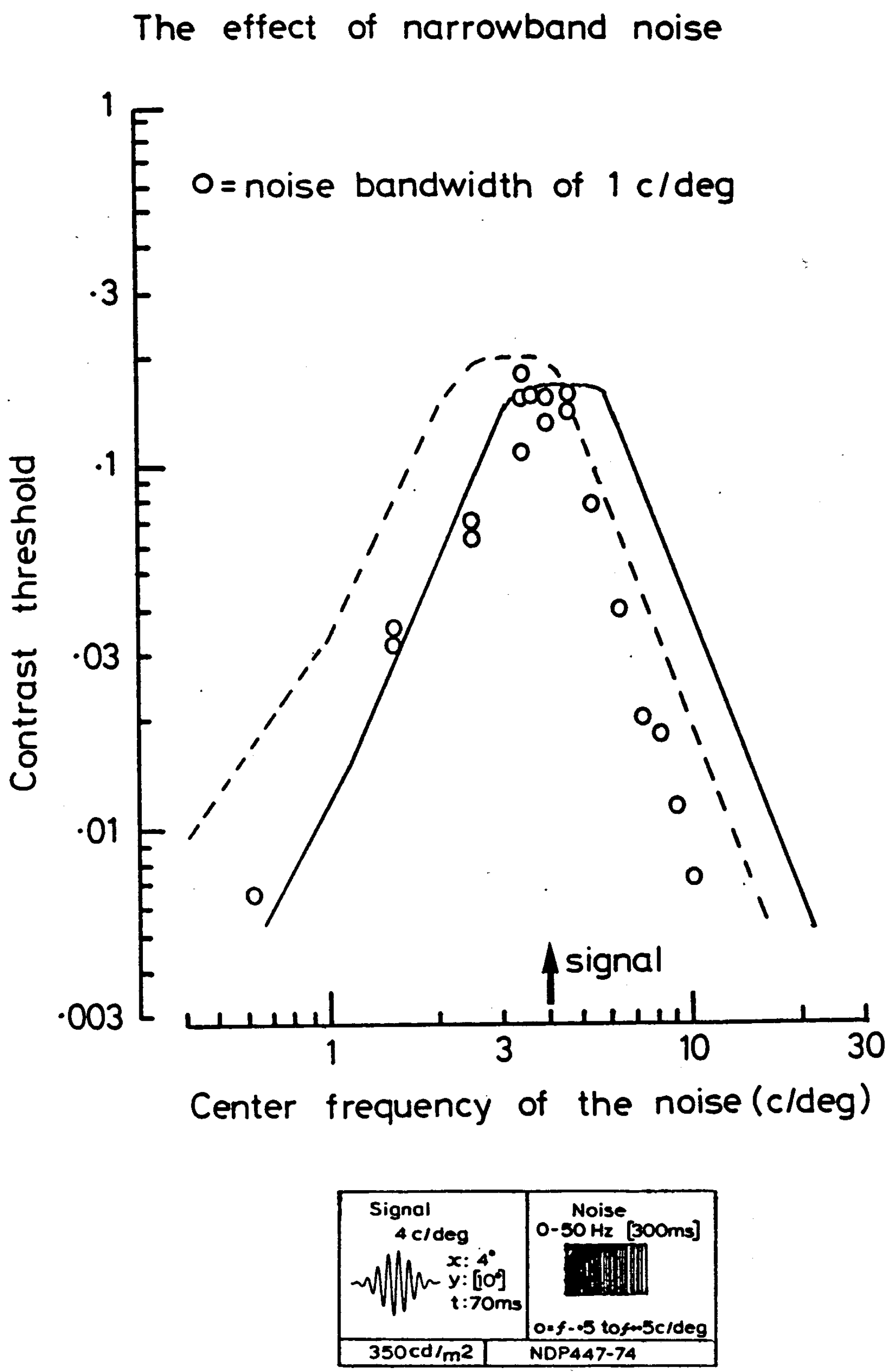

The effects of high-pass
and low-pass noise.
x = high-pass noise
o = low-pass noise
Contrast threshold
1
·3
·1
·03
·01
·003
signal
0
·3
1
3
10
30
Cutoff spatial frequency of the noise (c/deg
Signal
4c/deg
x: 4°
y: [10°]
t: 70ms
Noise
0-50Hz
o= 0-f c/deg
x= f-33 c/deg
350cd/m2
NDP502-37

Figure 3.11



The effect of narrowband noise
o = noise bandwidth of 1 c/deg
1
·3
·1
·03
·01
·003
Contrast threshold
signal
1
3
10
30
Center frequency of the noise (c/deg)
Signal
4 c/deg
x: 4°
y: [10°]
t: 70ms
Noise
0-50 Hz [300ms]
350cd/m2
NDP447-74

narrow-band-above-signal-frequency thresholds are almost a factor of 2 lower than the prediction. The appearance of the graph suggest that the observer may have used a channel with an even lower center frequency.

TEMPORAL TUNING

So far we have examined spatial-frequency tuning in both spatial dimensions. This section investigates temporal frequency tuning. The data were collected in collaboration with A.B. Watson in 1979.

Method

The most analogous technique to that of the previous sections would be to use one- or two-dimensional dynamic white noise, bandlimited to a range of temporal frequencies. However, this was not technically possible with the equipment available to us. Therefore we adopted a simpler technique within our technical capabilities. The signal parameters and observer (AB) were the same as in Watson and Robson (1981, they called AB, "ABW") to allow comparison with the temporal tuning they derive from frequency discrimination.

The signal was a 1/4 c/deg grating, and the noise was a randomly flickering 1/4 c/deg grating. The creation of both signal and noise began with a "master" 1/4 c/deg grating with a Gaussian envelope over space (6° radius to 1/e) and time (250 ms to 1/e).

$$\text{master}(x,y,t) = \sin(2\pi \tfrac{1}{4} x)\ e^{-(x/6°)^2-(y/6°)^2-(t/250\text{ms})^2}.$$

The signal was an attenuated version of this master, temporally modulated at either 0, 4, or 16 Hz,

$$\text{signal}(x,y,t) = c \cos(2\pi f_t t)\ \text{master}(x,y,t)$$

The noise was the product of the master and a random temporal modulation, r(t),

$$\text{noise}(x,y,t) = r(t)\ \text{master}(x,y,t).$$

The modulation, r(t), was bandlimited white noise. In the terminology of Appendix 1, the noise was dynamic and zero-dimensional because it varied randomly only over time. The "all-pass" white noise referred to in the text extended from 0 to 40 Hz. Low-pass and high-pass noise were derived by filtering the "all-pass" temporal noise prior to modulation of the 1/4 c/deg master.

The generation of the stimuli may be described in several steps. The computer synthesized the master pattern via a digital-to-analog converter. To produce the signal the master pattern was multiplied by the appropriate temporal factors (via a multiplying digital-to-analog converter) and attenuated. The (programmable) attenuator controlled the contrast, c, and produced the temporal Gaussian envelope.

Production of the noise involved more steps. The computer synthesized temporally white noise (one sample per frame) which was low-pass filtered by an 8-stage filter with cut-off at 40 Hz (which was below the 50 Hz Nyquist frequency). This was then either low- or high-pass filtered by another 8-stage filter. This bandlimited noise was sampled (via an analog-to-digital converter) once per frame. To produce the displayed noise, the master pattern was multiplied by the current noise sample (via a multiplying digital-to-analog converter) and attenuated. The (programmable) attenuator controlled the noise level and produced the temporal Gaussian envelope.

Finally the signal and noise were added and sent to the display.

The entire apparatus was carefully calibrated and checked for artifacts. Five alternate arrangements had previously been rejected because of artifacts which we ultimately traced to substantial feedthrough from the digital input of the multiplying digital-to-analog converters (MDACs) to their output, even when 0 volts was presented to their analog input. This characteristic is not usually specified by manufacturers of MDACs, so we eliminated the problem by changing the digital input to the MDAC only between frames.

Results

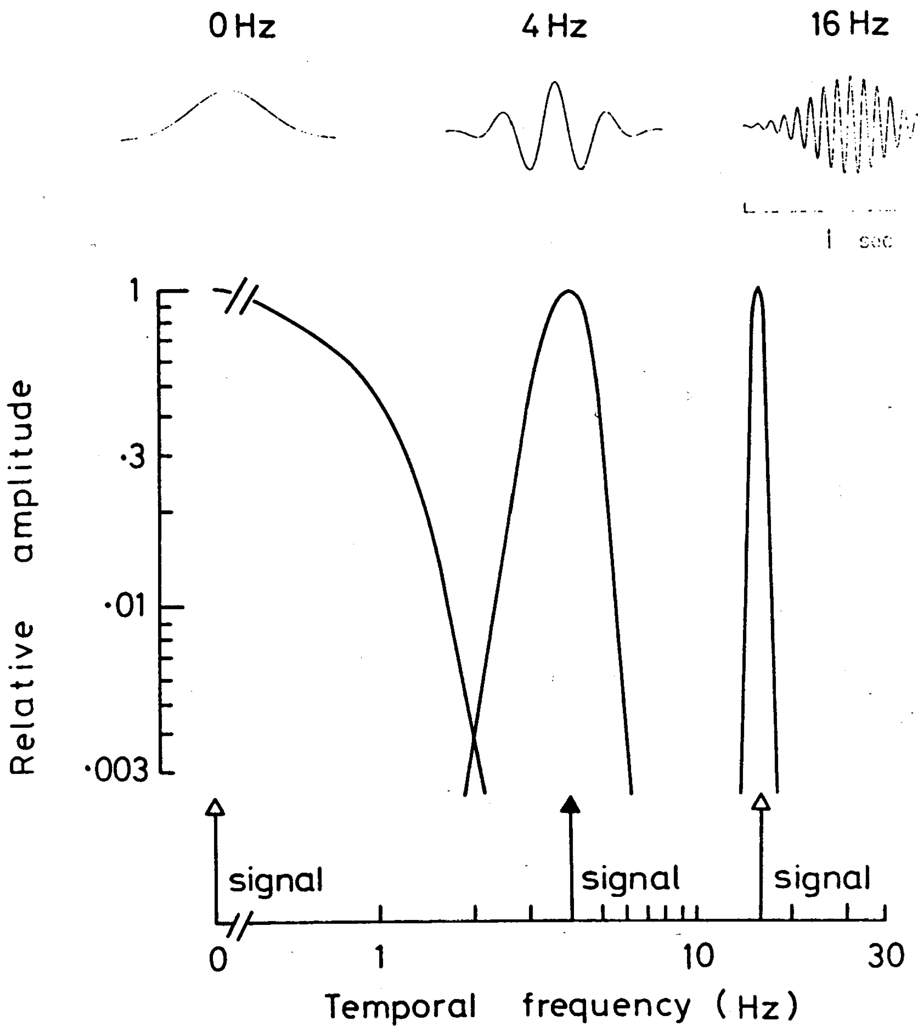
Figure 3.12 illustrates the temporal characteristics of the signals. Each temporal envelope is shown above its Fourier transform. The Gaussian envelope (with 250 ms time constant) of the stimuli determines their bandwidth at about 2 Hz full width at half height, except for the 0 Hz signal whose bandwidth is half that.

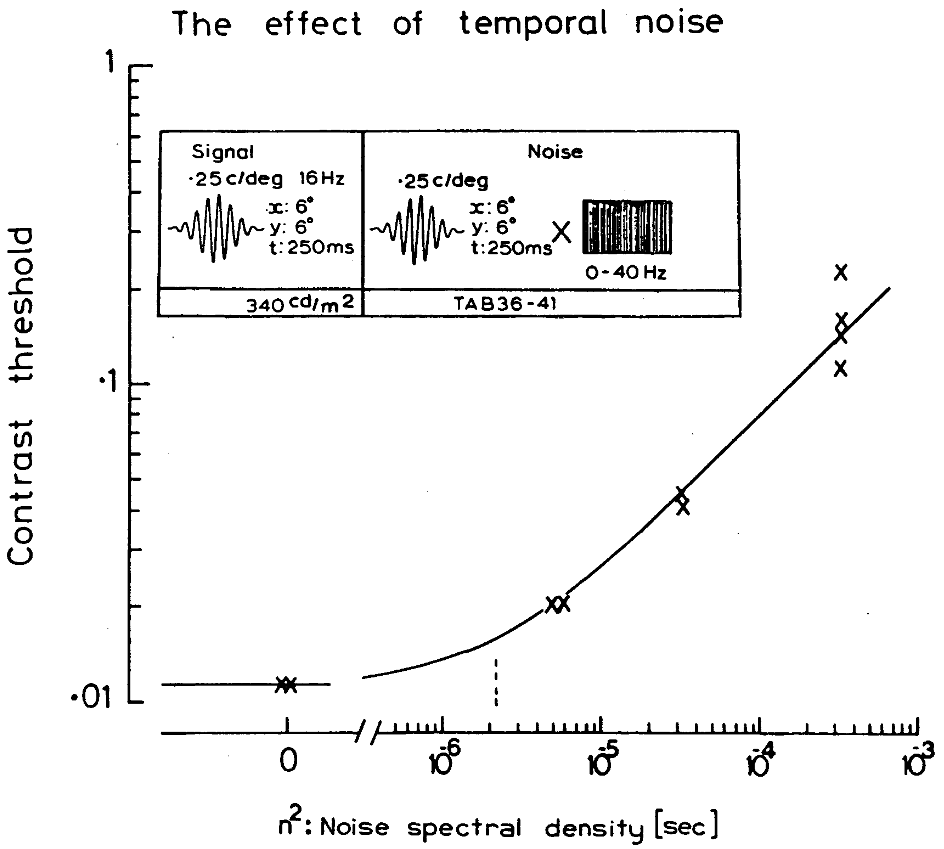
Figure 3.13 plots threshold versus noise spectral density of white (0-40 Hz) noise. The curve shows that squared threshold is proportional to the critical plus actual spectral density of the noise.

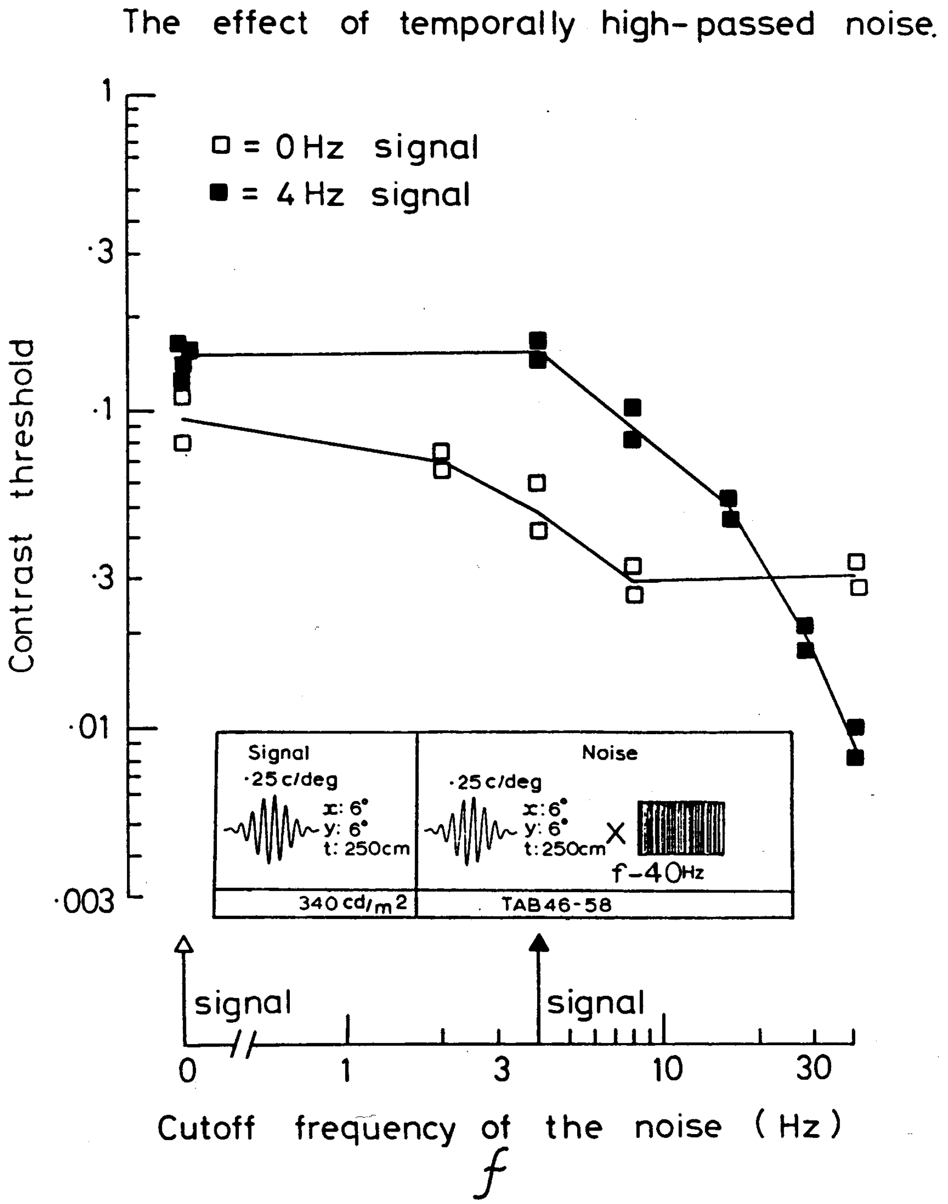
Figure 3.14 shows thresholds for 0 and 4 Hz signals in high-pass noise. Figure 3.15 shows thresholds for 4 and 16 Hz signals in low-pass noise. In each case the two signals yield distinct curves, not vertically displaced copies of each other, indicating that the signals are detected by different channels.

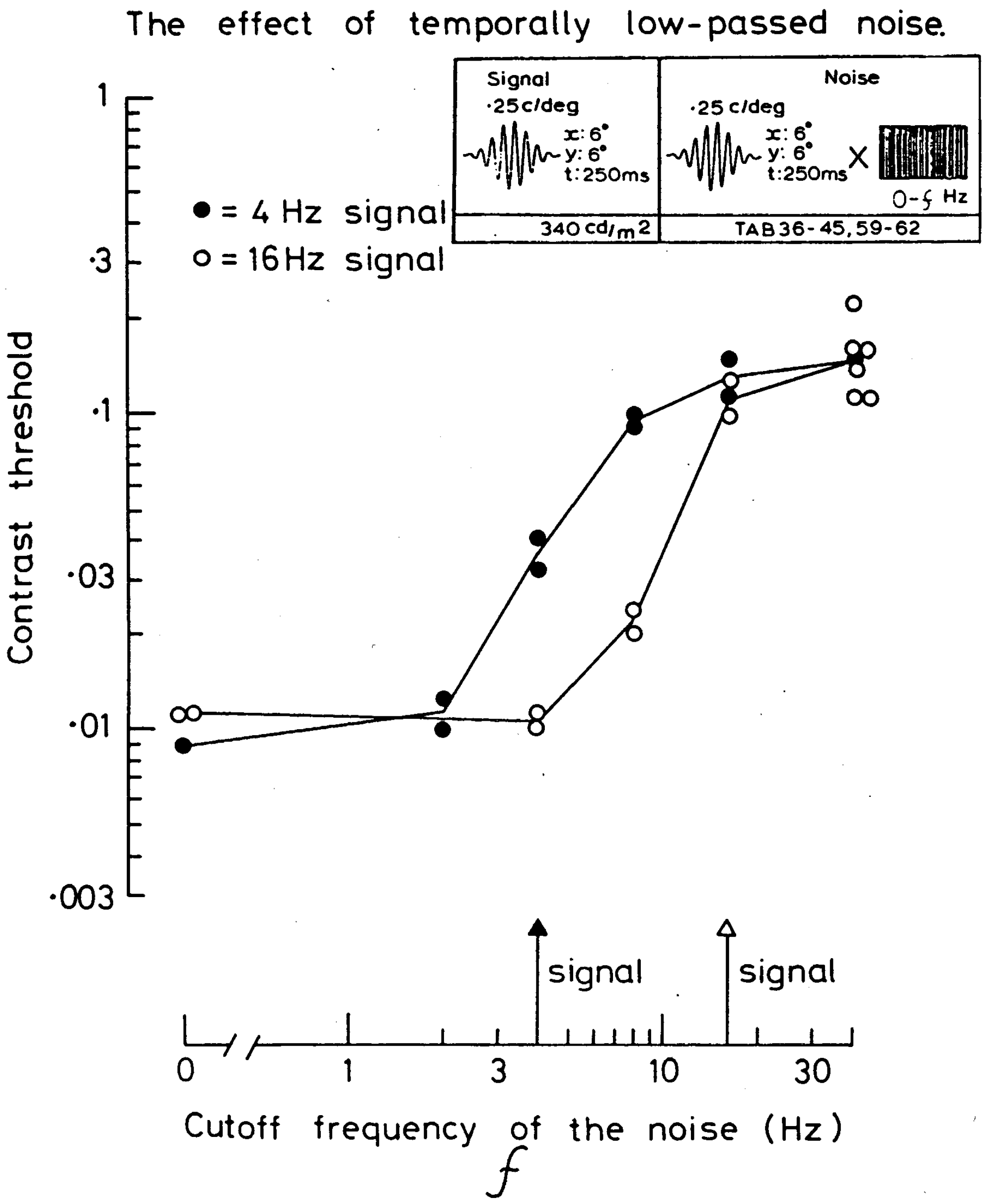
According to the total-filtered-noise-power hypothesis, the squared threshold due to narrowband noise, e.g. from 2 to 4 Hz, is the difference in squared thresholds in high-pass (or low-pass) noise due to changing the cut-off

The temporal envelopes and their amplitude spectra.

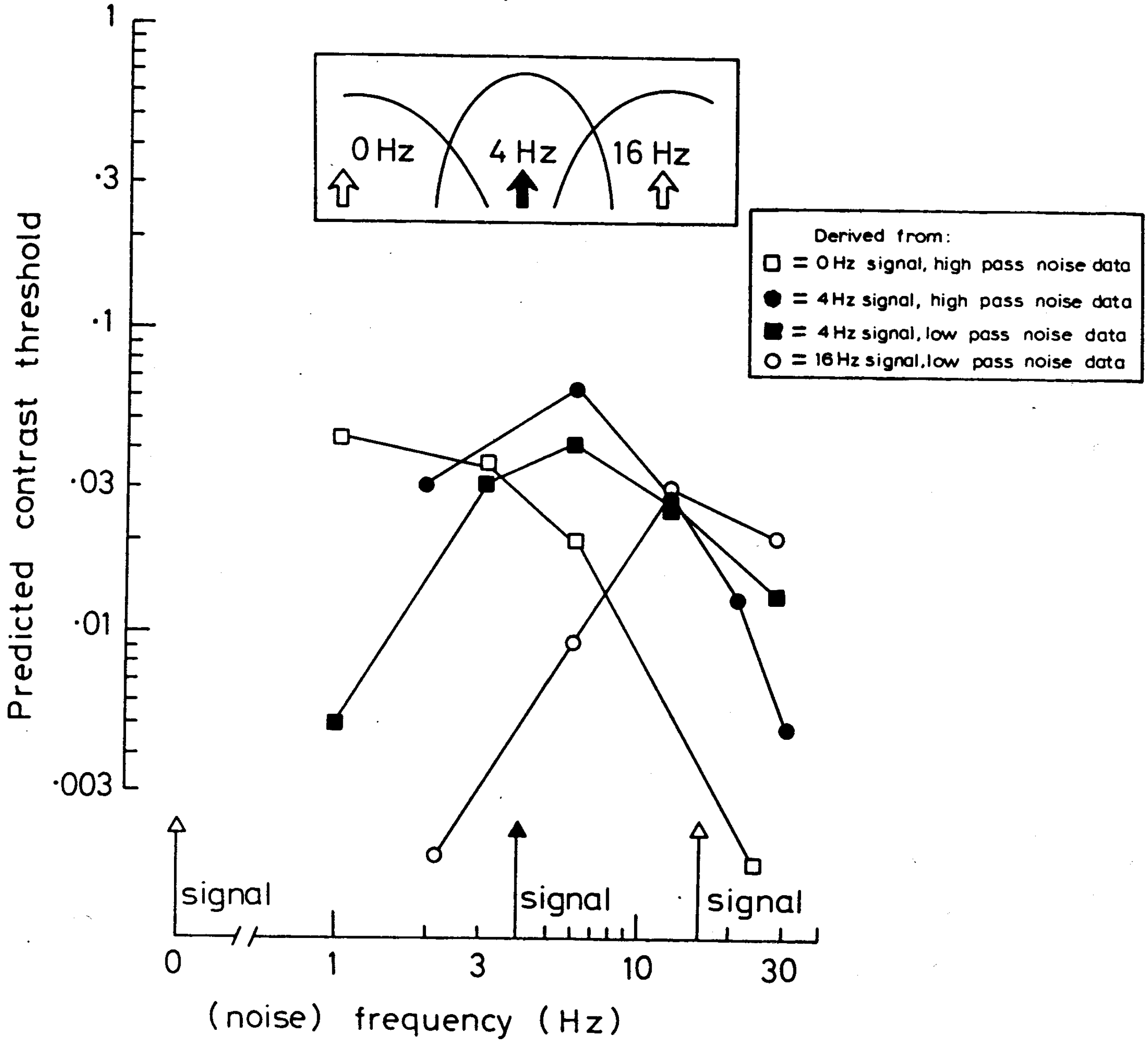



The effect of temporal noise
Signal
·25 c/deg 16 Hz
x: 6°
y: 6°
t: 250ms
Noise
·25 c/deg
x: 6°
y: 6°
t: 250ms
0-40 Hz
340 cd/m2
TAB36-41
Contrast threshold
1
·1
·01
0
10-6
10-5
10-4
10-3
n2: Noise spectral density [sec]


The effect of temporally high-passed noise.
□ = 0 Hz signal
■ = 4 Hz signal
Contrast threshold
1
·3
·1
·03
·01
·003
Signal
·25 c/deg
x: 6°
y: 6°
t: 250cm
Noise
·25 c/deg
x: 6°
y: 6°
t: 250cm
f-40Hz
340 cd/m2
TAB 46-58
signal
signal
0
1
3
10
30
Cutoff frequency of the noise (Hz)
f


The effect of temporally low-passed noise.
Signal
·25 c/deg
x: 6°
y: 6°
t: 250ms
Noise
·25 c/deg
x: 6°
y: 6°
t: 250ms
X
0-f Hz
340 cd/m2
TAB 36-45, 59-62
● = 4 Hz signal
○ = 16 Hz signal
Contrast threshold
1
·3
·1
·03
·01
·003
signal
signal
0
1
3
10
30
Cutoff frequency of the noise (Hz)
f

Estimated channel sensitivities (predicted effect of narrowband noise).



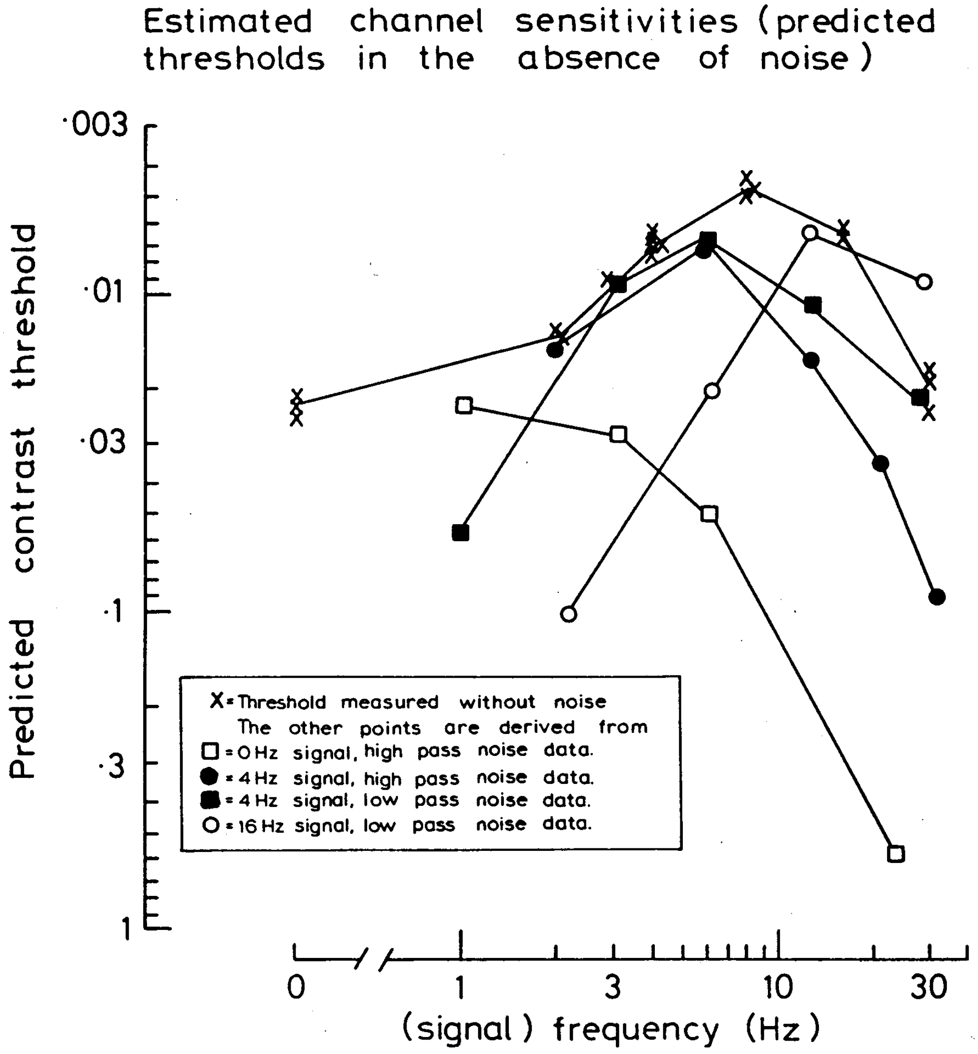
frequency by the appropriate amount (in our example, from 2 to 4 Hz). Given the total-filtered-noise-power hypothesis, narrowband noise thresholds are proportional to the channel's MTF.

Figure 3.16 shows these predicted thresholds in narrowband noise, except that, since the cut-off frequencies were not evenly spaced, the plotted threshold predictions are all for narrowband noise with the total power corresponding to that of 1 Hz band at the spectral density used in Figures 3.14 and 3.15. Each signal frequency, 0, 4, and 16 Hz, produced a distinct MTF, peaked near the signal frequency. There is fair agreement between the two MTFs for the 4 c/deg signal derived independently from low- and high-pass noise data. This indicates that the observer used the same channel in both high- and low-pass noise; he did not look off-frequency.

To compare with Watson and Robson's (1981) results we must deduce channel sensitivities in the absence of luminance noise. Figure 3.17 shows zero-noise thresholds and each MTF from Figure 3.16 has been adjusted vertically to fit the zero-noise thresholds.

Watson and Robson measured the ability of observers to discriminate the temporal frequency of near-threshold patterns. They found that observer AB ("ABW") could "perfectly discriminate" 0 Hz from 8 Hz, and 2 Hz from 16 Hz, but that 4 Hz was not perfectly discriminated from any frequency from 0 to 32 Hz. Watson (personal communication) has calculated that "labeled" channels should allow perfect discrimination when each one's threshold for its favored stimulus is a factor of 2 (i.e. 6 dB) less than its contrast threshold for the other stimulus. Figure 3.17 shows that the 4 Hz channel has fallen by more than a factor of two at 16 Hz and at 0 Hz (by extrapolation from 1 Hz), and that the 0 and 16 Hz channels have fallen by nearly a factor of ten at 4 Hz. Thus Figure

3.17 suggests that 4 Hz should be perfectly discriminated from 0 Hz and 16 Hz, contrary to Watson and Robson's finding for the same observer and signals. Since both experiments deduce channel tuning from frequency discrimination it is surprising that they seem to disagree.


Estimated channel sensitivities (predicted thresholds in the absence of noise)
·003
·01
·03
·1
·3
1
Predicted contrast threshold
0
1
3
10
30
(signal) frequency (Hz)
X = Threshold measured without noise
The other points are derived from
□ = 0 Hz signal, high pass noise data.
● = 4 Hz signal, high pass noise data.
■ = 4 Hz signal, low pass noise data.
○ = 16 Hz signal, low pass noise data.

SUMMARY

The chapter began by a critical examination of two hypotheses: the hypothesis that, for a given filter (i.e. channel), squared threshold is proportional to total filtered-noise power, and the one-signal-one-filter hypothesis.

Firstly, while squared threshold is proportional to spectral density of white noise (as shown in Chapter 2), and narrowband noise (shown here), the effect of bandwidth of narrowband noise centered on the signal frequency constitutes an important test. According to the total-filtered-noise-power hypothesis, squared threshold is proportional to the product of the spectral density and the bandwidth of narrowband noise, whereas, for example an energy detector's squared threshold is proportional to the product of the spectral density and the square root of the bandwidth of narrowband noise. Results of mine and Stromeyer and Julesz (1972) indicate that squared threshold rises in proportion to the spatial-frequency bandwidth of narrowband noise, up to a critical bandwidth of ±1 octave (2.5 to 10 c/deg about their 5 c/deg signal). A similar result is reported for the spatial frequency bandwidth orthogonal to the signal frequency: squared threshold is proportional to the orthogonal spatial-frequency bandwidth of two-dimensional dynamic noise up to a critical bandwidth of .6 c/deg for signal frequencies of 1, 3, and 9 c/deg. Thus both for variation of noise level and bandwidth (in both spatial frequency dimensions), squared threshold is proportional to the total power of narrowband noise.

Secondly, the filter MTF as a function of horizontal spatial frequency was derived, independently, from thresholds in high-pass, low-pass, and narrow-band noise. The three derived MTFs are all different, contradicting the one-signal-one-filter hypothesis.

Similar problems in audition led to the "off-frequency listening" hypothesis which supposes that a listener may use channels centered at frequencies away from the tone frequency in order to maximize his signal-to-noise ratio and thus lower his threshold for the signal. The vision data described above are compatible with an analogous off-frequency looking hypothesis and indicate that the observer learned to use channels centered at either 3.5 or 4.5 c/deg to detect a 4 c/deg signal, and thus reduce his threshold by as much as 4 dB. The high-pass, low-pass, and narrow-band noise experiments are all accounted for by two inferred filters each with a bandwidth of 1.5 to 2 octaves (full width at 3 dB point). The channel shapes were symmetric with steep skirts (log-log slopes of ±2.5).

An experiment designed to determine whether spatial frequency channels form discrete populations found that signal frequencies of 4, 3, and 2.5 c/deg all yielded channels with distinct center frequencies, suggesting that channels are densely distributed in this frequency range.

In collaboration with A.B. Watson, low and high-pass temporal noise modulation were used to study the temporal tuning for detection of a .25 c/deg pattern. The derived filter shapes are narrower than suggested by Watson and Robson (1981) on the basis of their frequency discrimination results. Three signal frequencies (0, 4, and 16 Hz) were used, and each yielded a distinct channel shape. Thus at .25 c/deg there are at least three channels with different temporal tuning functions. Low- and high-pass noise results yielded the same MTF for detection of the 4 Hz signal, indicating that off-frequency looking did not occur along the temporal-frequency dimension.

CHAPTER 4

EFFECTS OF NOISE-MASKING AND CONTRAST ADAPTATION ON CONTRAST DISCRIMINATION AND APPARENT-CONTRAST MATCHES

EFFECTS OF NOISE-MASKING AND CONTRAST ADAPTATION ON CONTRAST DISCRIMINATION AND APPARENT-CONTRAST MATCHES*

ABSTRACT

Chapters 1, 2, and 3 have examined effects of noise on detection. This chapter reports effects of noise on contrast discrimination and apparent-contrast matches. These tasks can be performed at all contrasts, not just near threshold. Near-threshold phenomena were similar at all noise levels, scaling with threshold. Suprathreshold phenomena were unaffected by noise. The notion of the observer's equivalent noise, which has been so useful in understanding detection, is found not to be relevant at suprathreshold contrasts.

Since noise masking and contrast adaptation both involve exposure to high rms contrasts (though vastly different spectral densities), and since both cause substantial threshold elevation, it is reasonable to ask whether they are related. It is found that neither affects suprathreshold contrast discrimination, but noise masking leaves suprathreshold apparent contrast matches unchanged, while contrast adaptation severely attenuates apparent contrast. These and other differences lead to the conclusion that noise masking is not a form of adaptation, though the converse may be true; the similar effects on contrast discrimination are consistent with the possibility that adaptation raises threshold by increasing the observer's equivalent noise.

INTRODUCTION

This chapter will investigate effects of noise on two tasks which can be performed at all contrasts, not just near threshold. We will see that white noise has virtually no effect on either suprathreshold apparent contrast matches or suprathreshold contrast discrimination. Near threshold the apparent contrast drops precipitously, and contrast discrimination has a characteristic facilitation.

It is well known that prolonged viewing of a high contrast grating results in a frequency selective elevation of threshold which lasts for many seconds (Pantle and Sekuler 1968, Blakemore and Campbell 1969). Since white noise is

* These results were presented at the 1979 meeting of the Association for Research in Vision and Ophthalmology, Sarasota, Florida (Pelli, 1979).

often used at similarly high rms contrasts (though vastly lower spectral density) it seems relevant to compare the effects of noise and adaptation to determine whether they might be considered different aspects of one phenomenon.

The proportionality of the squared threshold contrast to the sum of critical and actual spectral densities suggests that thresholds with and without external noise are similar phenomena. The only obvious difference is in appearance: the actual noise appears "noisy", while we are not usually aware of the equivalent noise, although some people claim to see it under some conditions (e.g. Fechner, Helmholtz, de Vries, 1943, Rose, 1948). Are there any other differences? This chapter uses two measures: contrast-discrimination thresholds and apparent-contrast matches.

Nachmias and Sansbury (1974) showed that we can discriminate near-threshold contrasts which differ by an amount less than the smallest contrast we can detect (i.e. discriminate from zero contrast). This is called "facilitation". They showed that there is a simple relation between the detectabilities and discriminabilities of contrasts. This finding will be called "d' additivity", and is discussed further in Chapter 5.

THE EFFECT OF NOISE ON CONTRAST DISCRIMINATION

In contrast detection an observer is asked to identify which of two intervals contains the pattern. In contrast discrimination the pattern is presented in both intervals at different contrasts, and the observer is asked to choose the interval with higher contrast. In this experiment the "signal" interval had a grating whose contrast was the sum of a pedestal and a difference contrast, the other interval had just the pedestal contrast.

Figure 4.1 plots the difference contrast required for 82% correct discrimination as a function of the pedestal contrast. The lowest curve connects the results collected in the absence of a noise mask. This replicates a similar curve of Nachmias and Sansbury (1974). The upper two curves (circles and X's) are the results when the experiment is done in the presence of a noise background. The threshold for detection (82% correct) at each spectral density is shown by an arrow on the abscissa.

The curves are very similar in two ways. The difference contrast falls as the pedestal is increased, reaching a minimum when the pedestal is approximately equal to threshold. When the pedestal is above threshold all the curves are equal: noise powerful enough to raise threshold more than tenfold did not affect contrast discrimination once the pedestal was above that threshold. The dip is of great theoretical interest because it indicates that the process of detection is similar with and without the noise mask.

The lack of any effect above threshold is of both practical and theoretical interest because it indicates that at least this characteristic of suprathreshold patterns is not affected by noise masking.

This result is similar to one obtained by Barlow, MacLeod, and van Meeteren (1976) using not noise masking, but contrast adaptation. The following experiment confirms this similarity of noise masking and contrast adaptation.

THE EFFECT OF ADAPTATION ON CONTRAST DISCRIMINATION

Method

This experiment is similar to the last except that threshold was raised by contrast adaptation instead of masking by noise. The lower curve in figure 4.2 shows the observer's difference thresholds before adaptation.

The effect of noise on contrast discrimination.

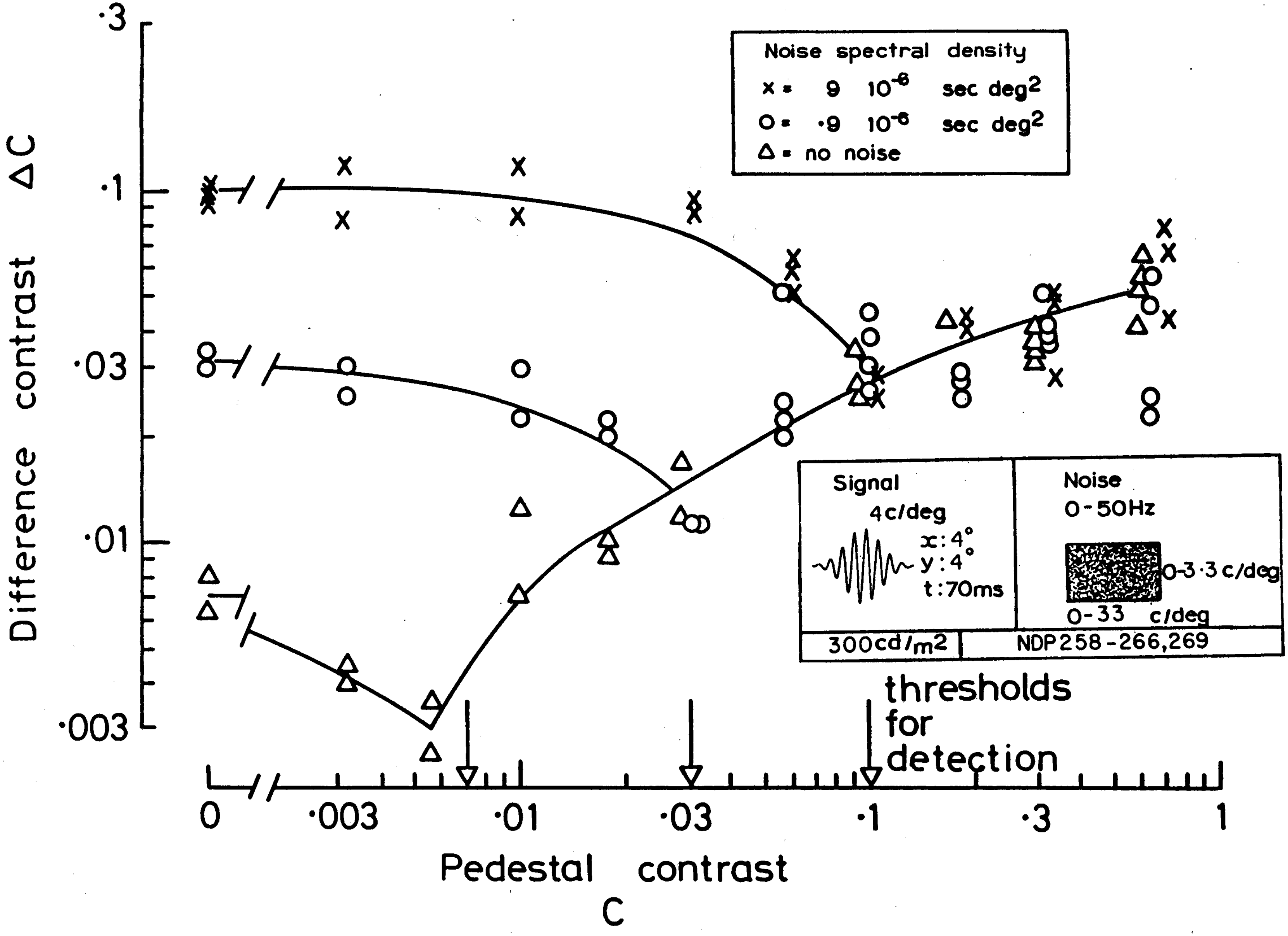


Figure 4.1 and 5.9b

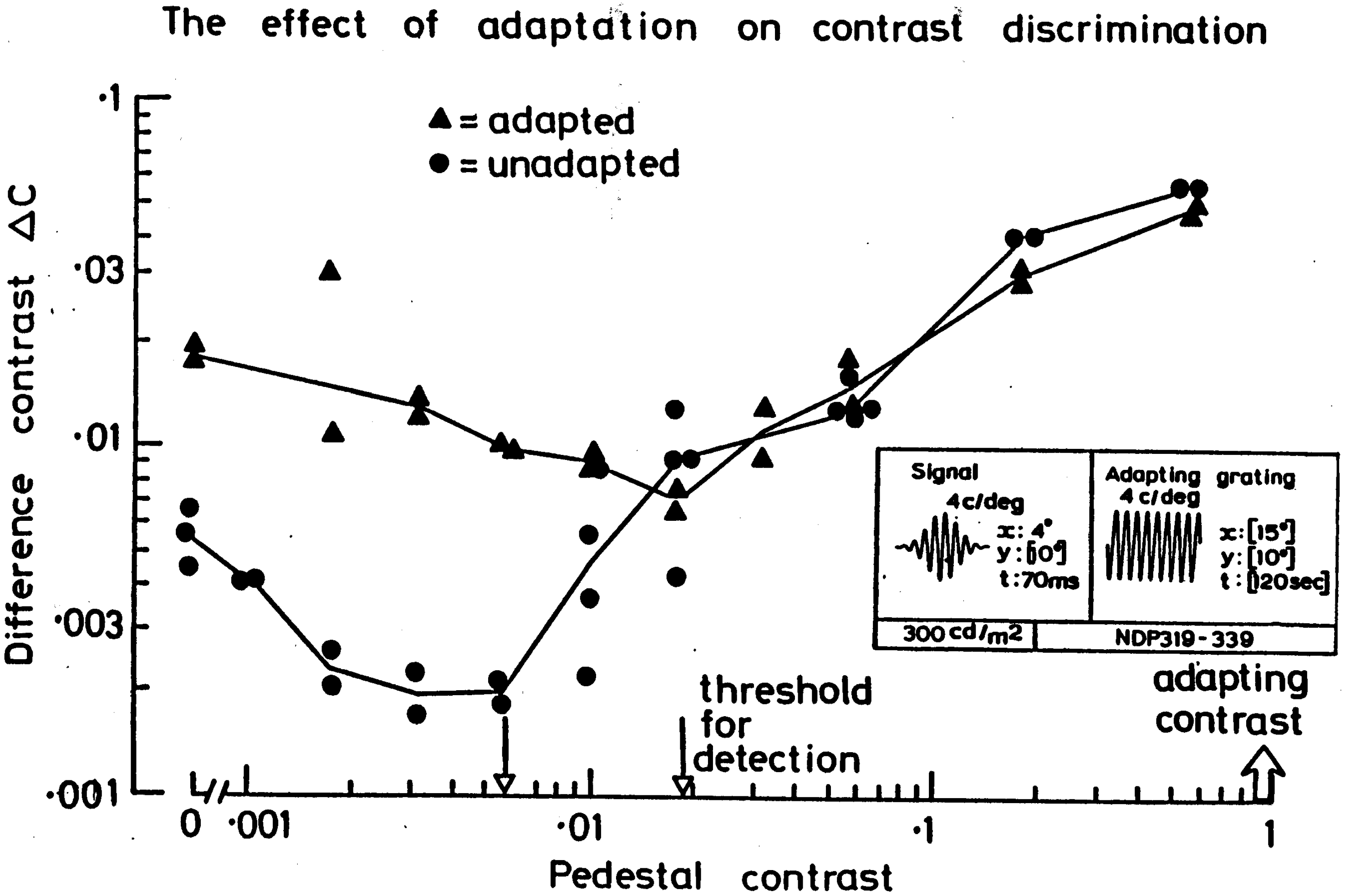

The effect of adaptation on contrast discrimination
▲ = adapted
● = unadapted
Difference contrast ΔC
·1
·03
·01
·003
·001
Signal
4c/deg
x: 4°
y: [10°]
t:70ms
Adapting grating
4 c/deg
x: [15°]
y: [10°]
t: [120sec]
300 cd/m2
NDP319-339
threshold for detection
adapting contrast
0
·001
·01
·1
1
Pedestal contrast

Figure 4.2

The observer adapted by viewing a .95 contrast grating for 2 minutes, as indicated in the inset in Figure 4.2. His adaptation was topped-up by viewing the .95 contrast grating for 3.5 seconds before each trial. The observer maintained fixation on a spot at the center of the screen throughout the experiment.

Adaptation raised threshold by a factor of 3. Difference contrast falls with increasing pedestal contrast until the pedestal reaches threshold; when the pedestal was above threshold the curves are equal. This latter finding confirms Barlow, MacLeod, and van Meeteren's (1976) report that contrast adaption does not affect supra-threshold contrast discrimination.

<u>THE EFFECT OF NOISE ON APPARENT-CONTRAST MATCHES</u>

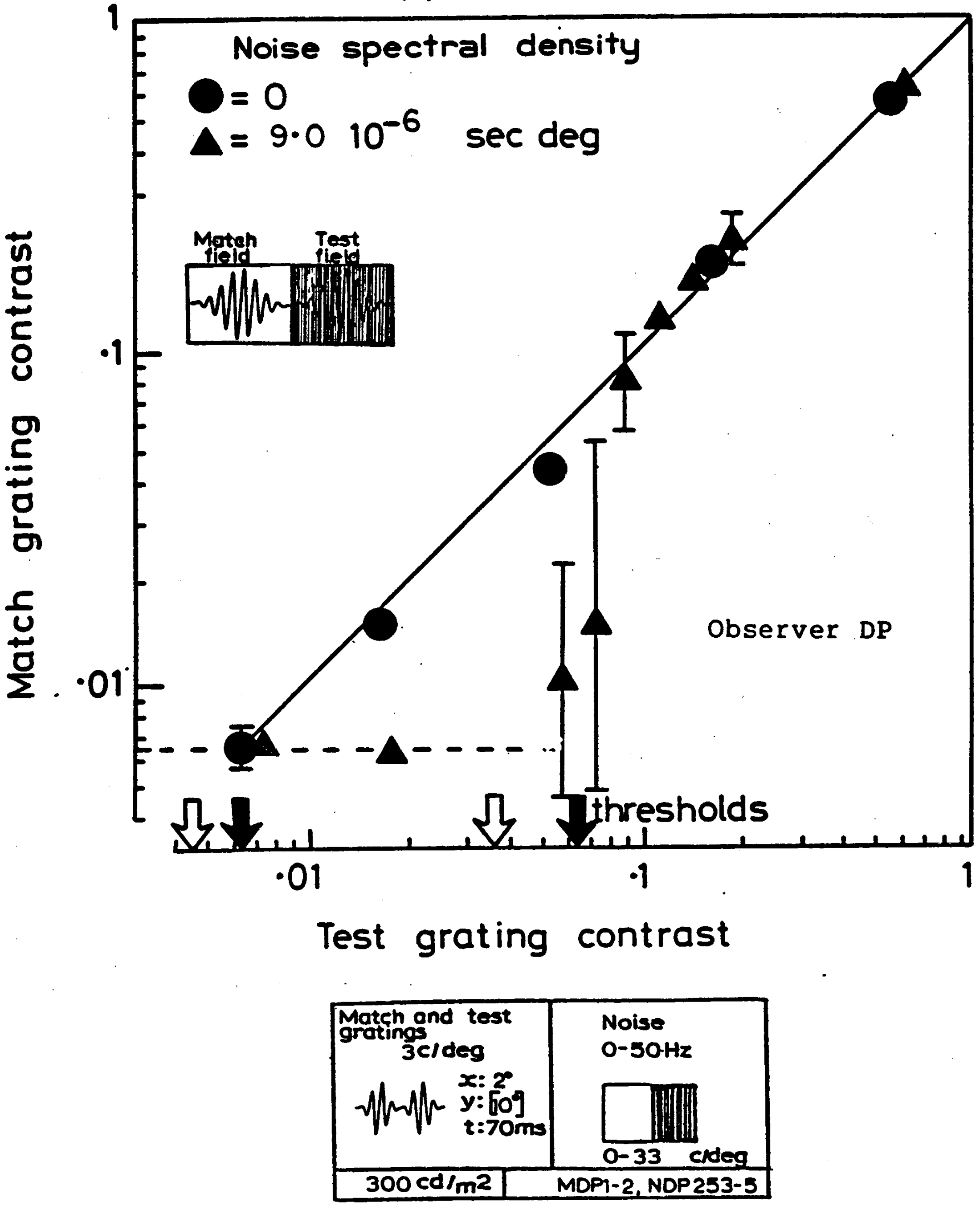
The display was split into two half fields. The test field contained a grating masked by noise. The observer was asked to vary the contrast of a grating in a match field so as to match the apparent contrast of the gratings in the two half fields. The log contrast was proportional to the angular position of a ten-turn potentiometer controlled by the observer plus a random offset which was changed after each match. Thus the observer had no manual clue to the physical contrast.

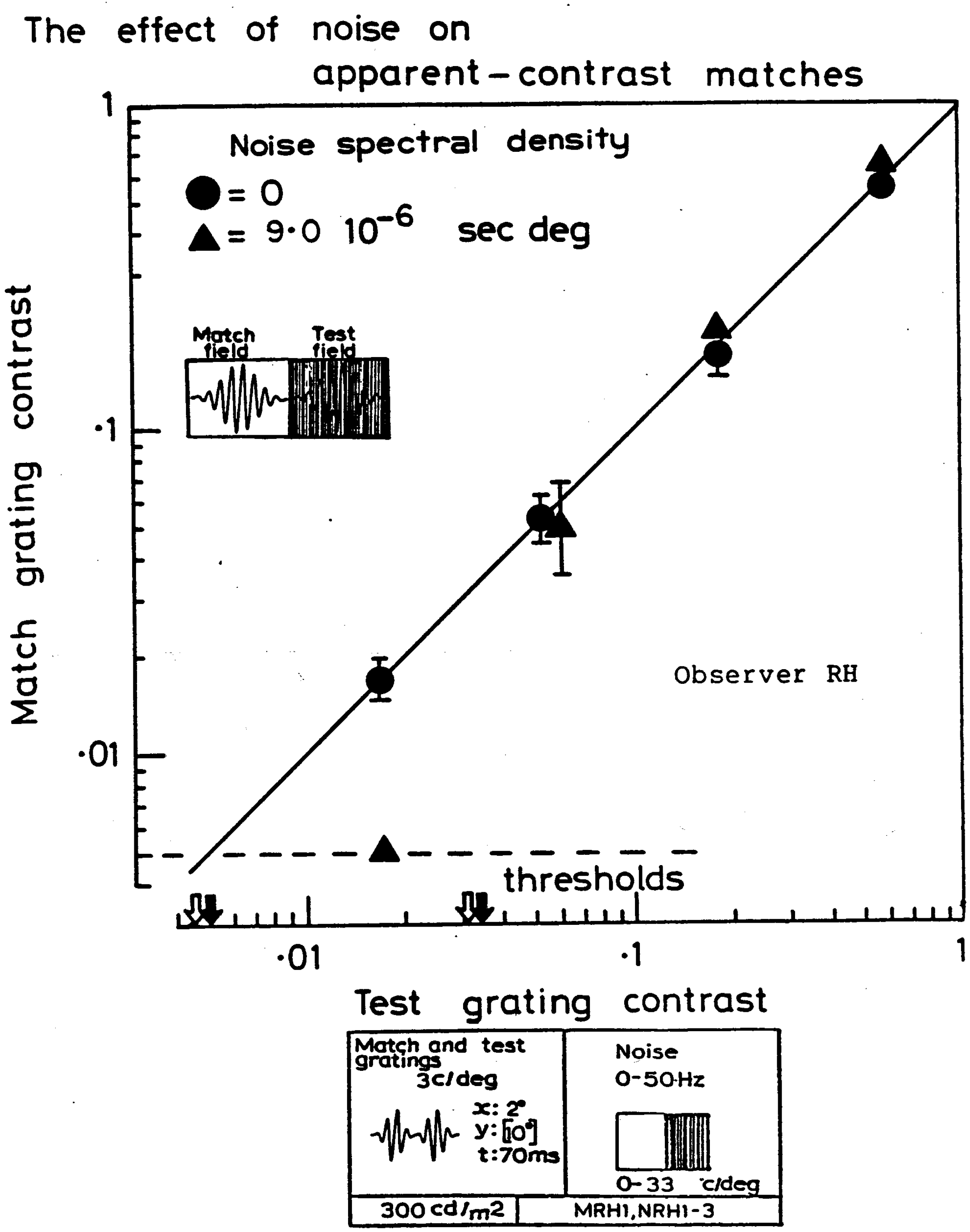
Figure 4.3 shows match contrast as a function of test contrast with and without a noise mask. Figure 4.4 shows similar results for a second observer.

When no noise was present the data (filled circles) all fall along the diagonal, as expected since the conditions were identical. When the test field had noise, subthreshold test contrasts were matched by subthreshold match contrasts (shown as filled triangles). The graph of match contrast as a function of test contrast rises very steeply near threshold and immediately joins the original diagonal representing a veridical match: the suprathreshold contrast matches were veridical.

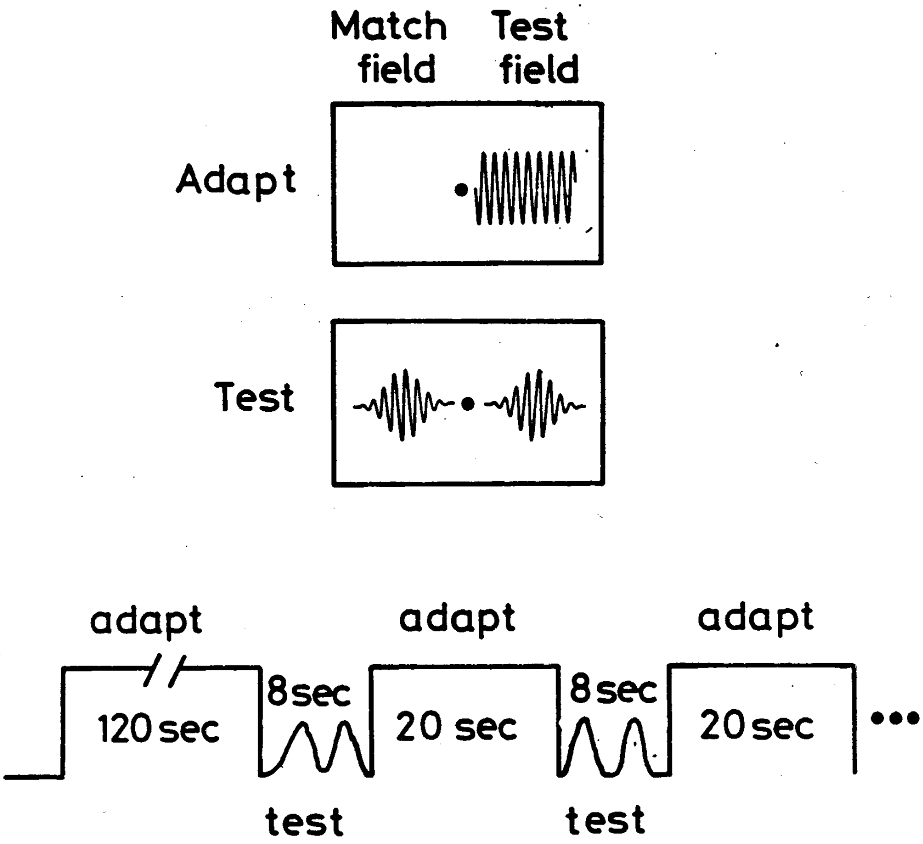
Figure 4.3

The effect of noise on apparent-contrast matches

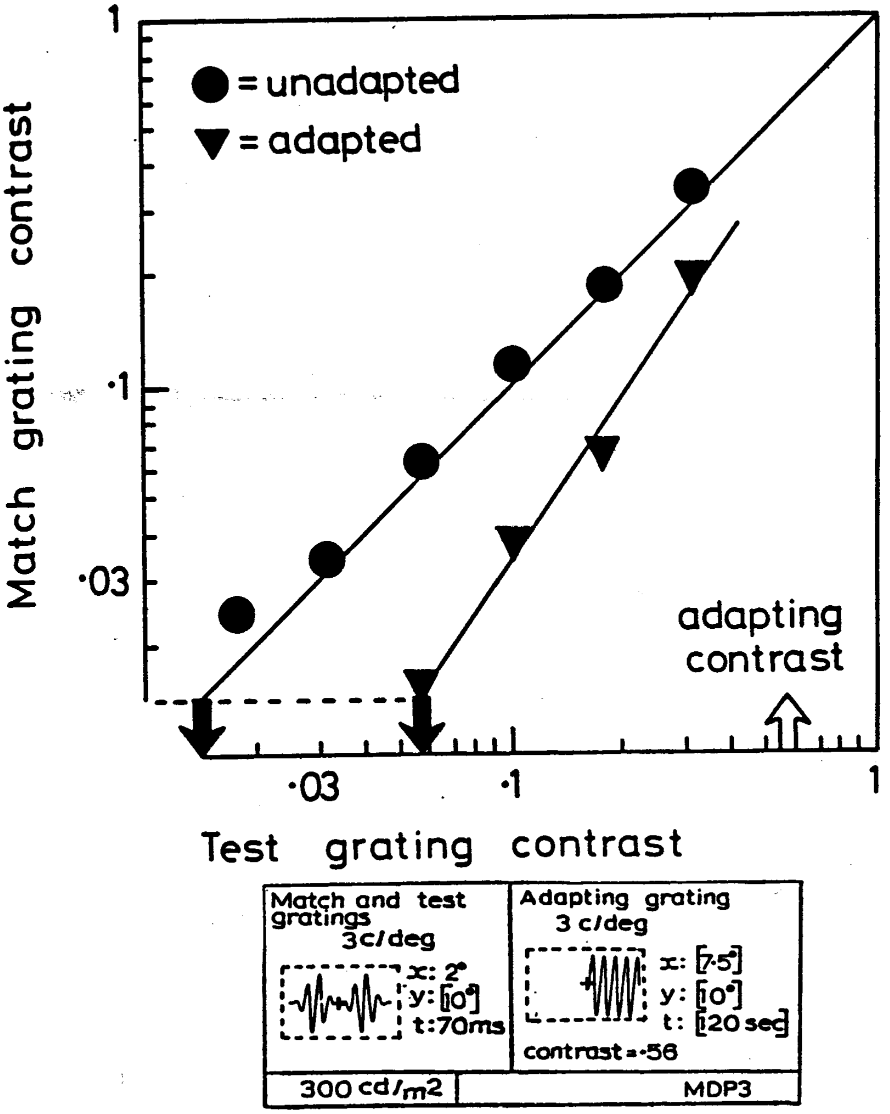



The effect of noise on apparent-contrast matches
Noise spectral density
● = 0
▲ = 9·0 10^-6 sec deg
Match field
Test field
Match grating contrast
1
·1
·01
Observer RH
thresholds
·01
·1
1
Test grating contrast
Match and test gratings
3c/deg
x: 2°
y: [10°]
t:70ms
Noise
0-50Hz
0-33 c/deg
300 cd/m2
MRH1,NRH1-3

The effect of adaptation on apparent contrast.



The effect of adaptation on apparent contrast matches.



Thresholds for detection were measured by the method of forced choice (82% correct 2AFC), shown as white arrows, and by the method of adjustment, shown as black arrows. The presence of the noise raised the forced-choice threshold fivefold, from .004 to .02, approximately, for both observers. DP's method-of-adjustment thresholds were approximately twice as high as RH's, despite their very similar forced-choice thresholds. The possibility of this sort of criterion difference was an important reason for using forced-choice as the primary method of threshold measurement.

The vertical bars indicate ±1 standard deviation about the mean log contrast of 2 to 8 settings. Where the bars are absent they were smaller than the plotted symbol. When the test grating was below threshold, and therefore invisible, the match contrasts were all below threshold. All below-threshold match contrasts were replaced with the method-of-adjustment-threshold contrast before calculation of the mean and standard deviation.

The sharp transition from matches with zero contrast to veridical matches was described by Georgeson and Sullivan (1975) for apparent contrast matches of gratings of unequal frequencies, and at unequal luminances. On matching apparent contrasts of unequal frequency they said,

> "The observers had some difficulty in making these matches because high frequency gratings seem to have high contrast as soon as they are above threshold. This is a paradox mentioned earlier: at threshold a high frequency grating is barely visible, yet may appear to be of higher contrast than the standard, which is easy to see."

THE EFFECT OF ADAPTATION ON APPARENT CONTRAST

This experiment is essentially a replication of one done by Blakemore, Muncey, and Ridley (1971), but using the same stimuli as in the noise-masking experiment: brief grating patches. Blakemore et al. used continuously present half-field gratings.

Method

Figure 4.5 illustrates the paradigm. The observer adapted to a grating in the test field with a contrast of .56 for two minutes. Then test and match gratings were presented repetitively for eight seconds. Each presentation of the gratings had the same brief temporal envelope as has been used up to now. During this period the observer was allowed to adjust the contrast of the match grating. When the observer was satisfied with the match he indicated the fact by pressing a button, and the computer recorded the match contrast, and then put up a new, randomly selected test contrast. The observer was allowed to take as long as he liked to make his match, but after every 8 seconds of matching, the adapting grating was returned to the test field to "top up" adaptation for another 20 seconds. After top-up the matching process was resumed automatically from where it was before the interruption. As before, the log contrast of a match grating was proportional to the position of the ten-turn potentiometer which the observer controlled, plus a random offset which was changed after each match.

Results

Figure 4.6 plots match contrast as a function of test contrast, in unadapted (shown as filled circles) and in the adapted state (shown as filled triangles). The unadapted data were collected in exactly the same way as the adapted data, except the adapting contrast was zero. Each point is the geometric mean of four settings. The standard deviation of the settings is approximately the diameter of a filled circle, except for the leftmost point which is a half of a standard deviation above the line. The lines were fitted by eye.

Despite the different stimuli and conditions used in this experiment, the results are very similar to those of Blakemore et al. Figure 4.6, as well as their results, shows that, in the adapted state, the log of match contrast is proportional to the log of test contrast: the points all fall on a straight line connecting threshold of match and test to unity contrast of match and test.

We have seen that noise masking and contrast adaptation produce very similar effects on contrast discrimination: discrimination with a sub-threshold pedestal exhibits facilitation, and discrimination with a suprathreshold pedestal is unaffected. This raised the possibility that noise masking and contrast adaptation had a similar mode of action. However, as we have just seen, suprathreshold matches of apparent contrast were reduced by adaptation and unaffected by noise masking.

CONCLUSIONS

Effects of luminance noise were compared with the effects of contrast adaptation. Noise masking elevates the threshold for detection without affecting suprathreshold contrast discrimination or apparent contrast. Contrast adaptation reduces the apparent contrast of all contrasts without affecting suprathreshold contrast discrimination. Thus they seem to be distinct phenomena.

The similarity of the effects of noise and contrast adaptation on contrast discrimination raises the possibility that contrast adaptation elevates threshold by increasing the observer's equivalent noise. Attempts to measure the observer's equivalent noise after contrast adaptation gave inconsistent results and are therefore not reported.

This Chapter's finding that near-threshold phenomena scale with threshold, combined with Chapter 2's evidence of proportionality of squared threshold to effective noise level, implies that near-threshold phenomena are unaffected by proportional scaling of squared contrast and effective noise level. Stated another way, these near-threshold phenomena are not affected by changing the amplitude of the effective stimulus. The theoretical consequences of this will be discussed in Chapter 5.

CHAPTER 5

UNCERTAINTY IN VISUAL DETECTION

UNCERTAINTY IN VISUAL DETECTION

ABSTRACT

We have seen that squared threshold is proportional to the effective noise level. This chapter seeks to incorporate this result into existing explanations of visual detection which succesfully account for other aspects of visual detection. In particular, the standard form of the "probability summation model successfully predicts summation effects - the increased detectability of extended patterns - from the steepness of the psychometric function for detection, and the "nonlinear transducer" model predicts contrast-discrimination performance also from the steepness of the psychometric function for detection. Neither of these models provides a satisfactory account of detection in luminance noise.

In detecting a signal an observer usually has two types of information available to him: what is presented during the trial, i.e. the stimulus, and the instructions and practice (often with feedback) he has received before the trial, i.e. prior information. Most attempts to understand visual detection have quite naturally emphasized the role of the stimulus and neglected the role of prior information. This chapter will consider the hypothesis that the observer is intrinsically uncertain, that is, that the observer is unable to make full use of prior information about the identity of the signal. Except for this limitation it is assumed the observer uses all information available in the ideal way. In order to make quantitative predictions it is necessary to make two simplifying assumptions (orthogonality and equal-detectability of "expected signals", and whiteness of effective noise) which yield the channel-uncertainty model.

The channel-uncertainty model predicts proportionality of squared threshold to effective noise level. It is known that this model can account for the steepness of the psychometic function, and that it is a "nonlinear transducer" model and a "probability summation" model in the most general sense of each term. It will be shown that the predictions of the channel uncertainty model do not differ sufficiently from the standard versions of probability summation and the nonlinear transducer models, in their intended areas of application, to be distinguished by existing experimental results. Thus the uncertainty hypothesis would seem to offer a parsimonious explanation of many aspects of visual detection.

Appendix 6 reports the results of a direct test ofthe uncertainty hypothesis. When the signal could appear at any of ten-thousand possible times and places over 20° and 2000 msec the observer's threshold was only 1.5-times higher than when the signal appeared at a fixed time and place on every trial. Clearly the observer made little use of exact information about the signal identity when it was provided, confirming the uncertainty hypothesis.

INTRODUCTION

Chapter 2 showed that quantum efficiency has two components, transduction efficiency and calculation efficiency, and found that the transduction efficiency of human observers shows little variation over a wide range of conditions. This chapter is concerned exclusively with calculation efficiency. It is assumed that the observer's equivalent noise has already been determined and referred to the visual field so that we may now ask how the observer makes decisions about that noisy effective stimulus (i.e. the contrast function of the actual stimulus, which may be noisy, plus the observer's equivalent noise).

REVIEW

Attempts to model human detection performance quite naturally first consider the ideal observer of a signal known exactly. For example Rose (1942,1948) and de Vries's (1943) fluctuation theory assumed that the observer would count photons in the area of the disc to be detected, and ignore the rest of the field. This is only possible if the size and location of the possible signal are known in advance.

d' expresses performance as the signal-to-noise ratio that would be required by an ideal observer (Tanner and Birdsall 1958).[1] Assuming an unknown, but constant noise spectral density, n^2, added to a known signal, the signal-to-noise ratio, s/n, is proportional to signal contrast (see the Introduction of Chapter 2). If the human observer acted like the ideal, d' would be proportional to signal contrast.

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[1] As noted earlier, this definition of d' is convenient in that it makes no assumptions about the observer. This chapter will refer to many works that used other definitions, however we will mostly be considering situations in which the definitions are numerically equal, so footnotes will appear only when the distinction matters.

Tanner and Swets (1954) found that both yes-no and four-alternative-forced-choice methods yielded similar estimates of d' for detection of a small spot on a uniform background. They found d' was not proportional to signal contrast. Instead they found d' proportional to approximately the third power of signal contrast. This result seems to be general: d' is proportional to the signal contrast raised to a power greater than 1. Tanner (1961) and Nachmias and Kocher (1970) confirmed this relation for detection of spots[2], and they also cite Blackwell's (1963) conclusion that the data reported in Blackwell (1946) indicate that d' rises as a power of the signal contrast greater than one. Nachmias and Sansbury (1974) and Foley and Legge (1981) extended it to detection of sinusoidal gratings.

Tanner (1961) suggested that the ideal-detection-of-a-signal-known-exactly model of human performance might be wrong in assuming exact knowledge of the signal:

> "An examination of our procedure indicates there might be uncertainty both as to the time and location of the signal presentation. . . . The uncertainty may be introduced at a central rather than retinal level, suggesting that one should not try to account for nonlinearities at the level of the end-organ, to the exclusion of central hypotheses."

Tanner noted that an ideal observer with uncertainty as to the signal could have the steep psychometric functions (i.e. $d' \propto c^k$, $k>1$) observed experimentally. Nachmias and Kocher (1970) noted that too, and the additional fact that their steep ROC curves might also be explained by signal uncertainty.

Cohn (1981) has shown, by simulation, that uncertainty would reduce quantum efficiency[3], and noted that this would help explain why our quantum

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[2] In rating scale experiments each point on the ROC curve yields a value for d' (by the definition we are using). Nachmias and Kocher (1970) calculated d' for the interpolated point with a 50% hit rate.

efficiency is lower than the estimates of photon absorption (Barlow 1977).

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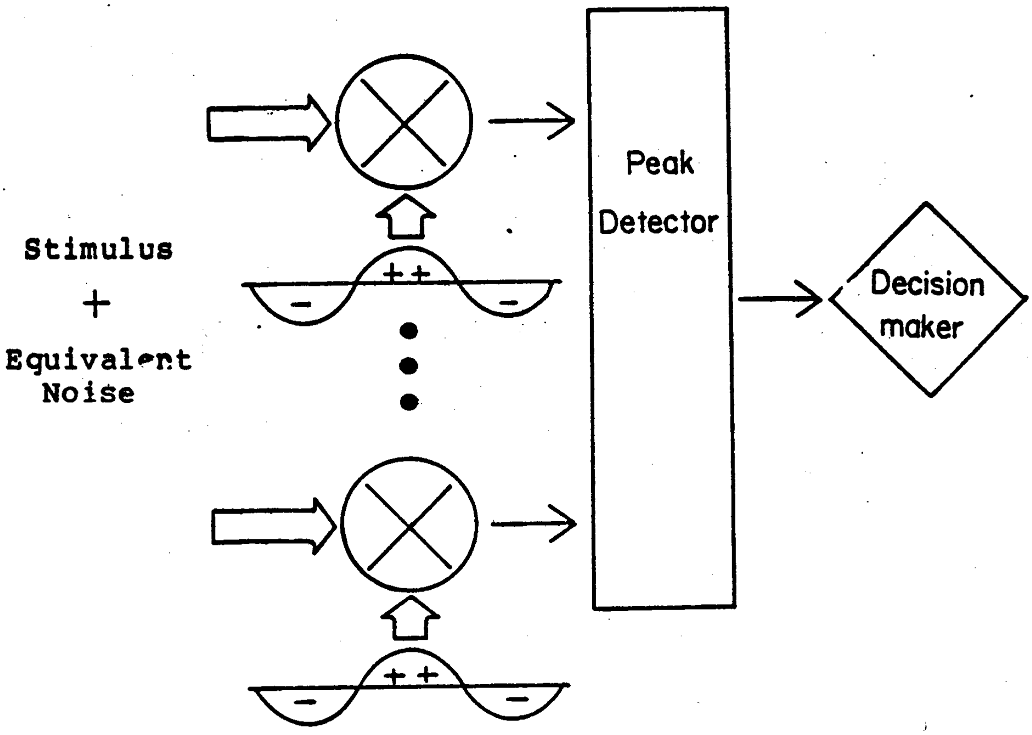
[3] This is what Cohn meant. He said uncertainty would "cause psychophysical estimates of quantum efficiency to be too low," but he was using a peculiar definition of quantum efficiency: "the fraction of available photons used in the task". This is meaningful as a parameter in his simple model (used in his simulations) but it is not at all clear what this would mean for something as complex as a human observer. A related point was discussed at the end of the Introduction to Chapter 2.

CHANNEL-UNCERTAINTY MODEL



THE CHANNEL-UNCERTAINTY MODEL

Theoretical background

Even the earliest papers on signal detection theory realized that it would often be unreasonable to assume exact knowledge of the signal to be detected. One of the alternative assumptions they considered was detection of one of M orthogonal signals. Peterson, Birdsall, and Fox (1954) showed that the ideal detector of one of many possible signals[4] would evaluate the likelihood of each signal and add the likelihoods to obtain the overall likelihood of a signal presence. Unfortunately the resulting equations are difficult to manipulate and Peterson et al. were only able to provide an approximate solution for the ideal's performance as a function of the number (M) and strength (i.e. the signal-to-noise ratio, s/n[5]) of the signals:

$$(d')^2 \approx \ln(1 - \frac{1}{M} + \frac{1}{M}e^{(s/n)^2})$$

Nolte and Jaarsma (1967) solved the original equations numerically and published receiver operating characteristic (ROC) curves for several values of M and s/n. They found that Peterson et al.'s approximation was accurate when s/n was less than 1, and very inaccurate when s/n was as large as 4. The interesting cases for the purposes of this chapter turn out to have large values of M (i.e. high uncertainty). When M is greater than 100 the approximation is inaccurate for values of d' greater than 0.5. Human psychophysical performance can be measured with reasonable accuracy and effort only over the range of d' extending from approximately 0.1 to 2, and the Peterson et al. approximation is inaccurate over most of this range. Nolte and Jaarsma's solutions are accurate but their paper could only include solutions for a modest number of pairs of values of M and s/n. Fortunately there is now a shortcut to the heroics of the direct solution which Nolte and Jaarsma undertook.

Nolte and Jaarsma also calculated the performance of a simple channel-uncertainty model which they called a "disjunctive receiver" (after Wainstein and Zubakov 1962). This receiver has a channel for each possible signal. Each channel evaluates the cross-correlation (i.e. integrated product) of the input (i.e. noise and perhaps a signal) with one of the possible signals. It is called a "disjunctive receiver" because each channel then applies a criterion to the cross-correlation and the receiver says "yes" if any channel exceeded criterion. The ROC curves

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[4] assumed orthogonal, equally-detectable, and added to a background of Gaussian white noise. Given the other two assumptions, equal-detectability is equivalent to assuming the signals have equal contrast energy.

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[5] The notation used by signal-detectability theorists has evolved much since Peterson, Birdsall, and Fox (1954). My notation follows Tanner and Birdsall (1958) except for signal energy (my s^2=their E) and noise power spectral density (my n^2=their $N_0/2^k$ where k is the dimensionality of the noise). Thus the signal-to-noise ratio was represented as $\sqrt{d_1}$ by Peterson et al., as $\sqrt{(2E_S/N_0)}$ by Tanner and Birdsall, and as s/n by me.

Nolte and Jaarsma plotted for this sub-optimal receiver are virtually identical (but not greater than) those of the ideal. The rest of this chapter will consider only a channel-uncertainty model which is equivalent to the disjunctive receiver for the yes-no detection paradigms Nolte and Jaarsma considered, but which will also behave sensibly in tasks such as contrast discrimination which we know human observers are capable of.

Following common practice it will be called a "channel uncertainty" model. However, the reader should be warned that these channels bear little relation to "spatial-frequency channels". One "spatial-frequency channel" could provide many cross-correlations (during the course of a psychophysical presentation), each of which would be attributed to a different cross-correlation channel in the context of this analysis.

The new model

A channel-uncertainty model was described by Wainstein and Zubakov (1962). As mentioned above, Nolte and Jaarsma showed that it performs nearly ideal detection of uncertain signals. Nachmias and Kocher (1970) proposed it as a model of visual detection. All these people considered only yes-no and rating-scale experiments, and their version of the model (i.e. the disjunctive receiver) cannot cope with a forced-choice paradigm. A more general version will be presented which copes with all the paradigms, and is equivalent to the older version for the yes-no and rating-scale paradigms.

This new model is illustrated in figure 5.1. The input is assumed to be a sum of the stimulus and the observer's equivalent noise. There are a number of parallel channels, each "expecting" a different signal. Each channel evaluates the cross-correlation of the input with the signal it is "expecting". For simplicity the expected signals are assumed to be orthogonal, and the effective noise is assumed to be Gaussian white noise. The assumptions are needed for two reasons. Firstly, the ideal has only been worked out for this case. Secondly, these assumptions imply that the various channels will be stochastically independent. Without independence the calculation of performance would be much more difficult, probably ruling out a direct numerical solution, and requiring Monte Carlo techniques. The orthogonality requirement would be satisfied by, for example, expected signals that do not overlap in space, or do not overlap in time, or that do not share any spatial frequencies. Each channel produces a single value for each stimulus interval. All these values go to the peak detector which passes only the largest one. Finally, the decision maker on the right receives the peak value for each stimulus interval. In two-alternative-forced-choice the decision maker will choose the interval which resulted in the larger peak value.

The assumption that the effective noise is white is plausible; Chapter 2's estimates of the observer's equivalent noise showed little dependence on test frequency, indicating that the equivalent noise is not far from being white.[6] The orthogonality assumption is not plausible, but I believe it is a reasonable simplification which may not seriously affect the solution. Imagine an array of visual neurons implementing the model. If the array is dense the receptive fields will overlap, and if it is very dense the receptive fields will not be orthogonal; their responses to white noise will be correlated. However the response of a cell correlated to its two neighbors will tend to be intermediate between their responses and thus rarely if ever provide the peak response, which is all that matters to the peak detector.

It is important to realize that the uncertainty may be over time. Returning to the visual neurons analogy, each neuron's output will vary over the course of the psychophysical presentation. If the cell is linear then several samples, separated sufficiently in time to be uncorrelated, would each

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[6] Of course, since the effective noise is a sum of the equivalent and actual noises, it could be made to have almost any spectrum. However this chapter will only consider experiments which used white luminance noise, or none at all.

be a cross-correlation of the input with a spatio-temporal weighting function, and each of the weighting functions (which are delayed copies of one another) would correspond to an "expected signal" in the theoretical development.[7]

The intent of this chapter is to explore the consequences of the uncertainty hypothesis. The simplifying assumptions of equal detectability, orthogonality, and white effective noise which lead to the channel-uncertainty model are needed to make the calculations feasible, and desirable in that they yield a model with only one degree of freedom: the degree of uncertainty, M.[8]

For ease of presentation only two-alternative-forced-choice contrast detection and discrimination of sinusoidal gratings will be examined. In such experiments the grating is presented at one contrast in the first interval, and at another in the second interval. The observer is asked to say which interval had higher contrast, and one can determine his proportion correct, P(C). That task is contrast discrimination. When the lower of the two contrasts is zero it is called contrast detection. Parallel existing results for other paradigms and signals will be cited where appropriate.

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[7] While it seems unlikely that a real peak detector would wait out the necessary delays to ensure that the successive values are uncorrelated, I suspect that the peak of samples taken at the maximum rate at which the samples would still be uncorrelated, would be similar to the peak over the continuous function.

---

[8] However the comparisons with human performance presented here will allow a second degree of freedom: the spectral density of the observer's equivalent noise. Chapter 2 showed that $n^2_{critical}$ is an estimate of the observer's equivalent noise, but this measurement was not available for the data considered here.

1. CONTRAST DETECTION

1a. Human

Figure 5.2 shows Nachmias and Sansbury's (1974) results for human performance at contrast detection. The abscissa is a logarithmic scale showing contrast of the grating to be detected, and the ordinate on the right shows the proportion of correct responses. The ordinate on the left is a logarithmic scale of d'[9]. The scale of the proportion-correct ordinate is determined by the logarithmic scale of the d' ordinate. Nachmias and Sansbury fit their data by the solid straight line with slope 2.9. It provides a good fit. For a second observer they reported a slope of 2.2. Thus, for both observers d' was proportional to the contrast raised to a power greater than 1.

Some readers may be more familiar with the Weibull (1951) function suggested by Quick (1974) to describe the psychometric function:

$$P(C)=1-.5e^{-\left(\frac{c}{c_{.82}}\right)^{\beta}}$$

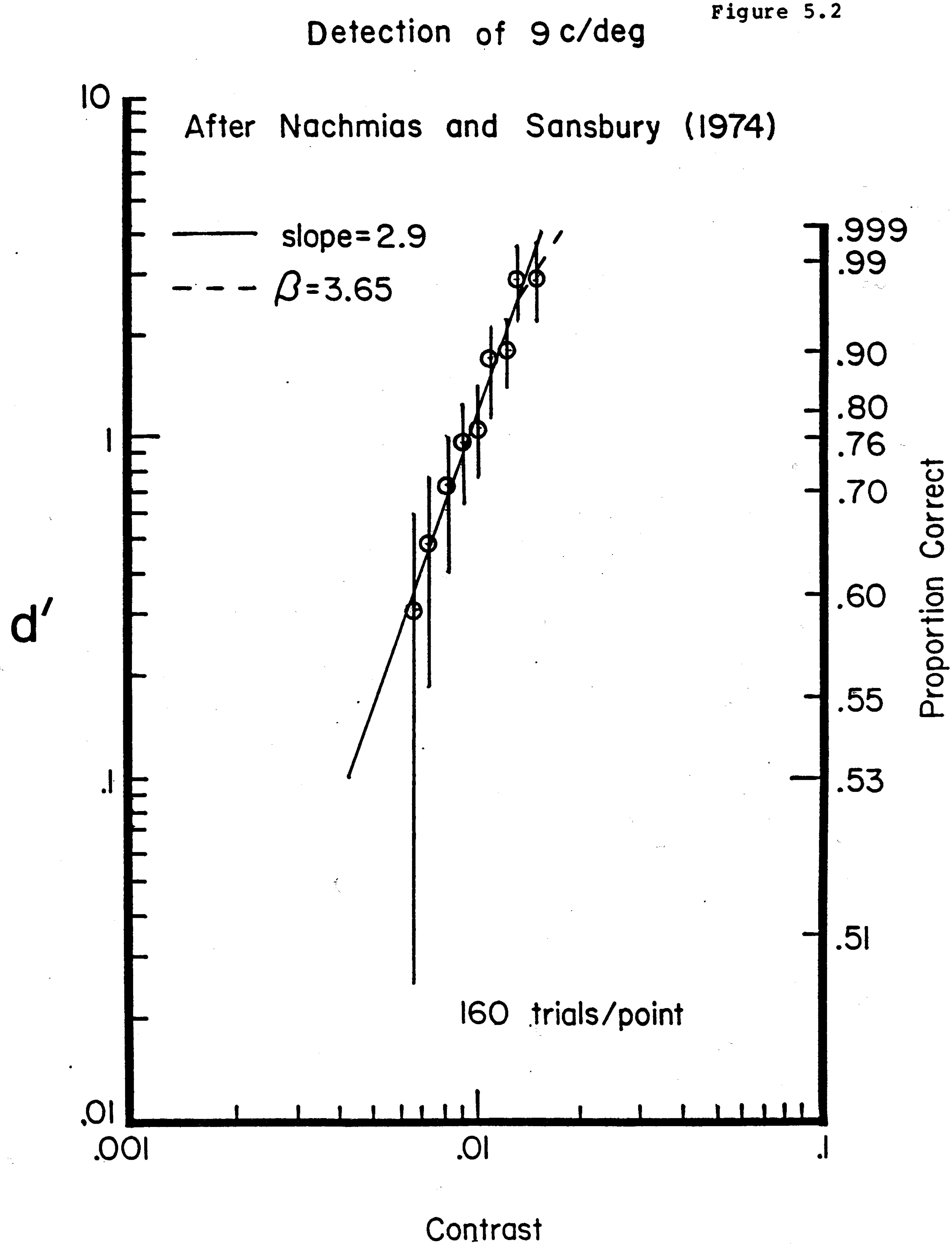
A maximum likelihood fit[10] by the Weibull function is shown too, as a dashed curve, but it is almost entirely covered by the solid line because, in these coordinates, the Weibull function produces a nearly straight line with a slope of approximately 0.83β.[11]

[9] d' is defined as the signal-to-noise ratio required by an ideal observer to perform as well as the observer under study (Tanner and Birdsall, 1958), but in the experiments reported here the signal was non-random, so d' is just $\sqrt{2}$ times the normal deviate corresponding to the proportion correct:

$$d'=\sqrt{2}\ \Phi^{-1}\{P(C)\}.$$

[10] The maximum likelihood fit was made by Watson's (1979) computer program, QUICK.

Figure 5.2


Detection of 9 c/deg
After Nachmias and Sansbury (1974)
slope=2.9
β=3.65
160 trials/point
d′
Proportion Correct
Contrast
10
1
.1
.01
.001
.01
.1
.999
.99
.90
.80
.76
.70
.60
.55
.53
.51

1b. Channel-uncertainty model

Theory

By assumption each channel evaluates the cross-correlation of the input with some function, in a sense the signal it is expecting. It is assumed that signals are orthogonal and have equal energy and that the noise is white and Gaussian. This implies that all channels produce values which are uncorrelated with equal variance, and with equal mean response to its "expected" signal. For convenience the output values are measured in units of the common standard deviation, so a channel responding to noise alone has probability, $\Phi(\lambda)$, of not exceeding a value λ, where $\Phi(\lambda)$ is the cumulative normal integral:

$$\Phi(\lambda)=\frac{1}{\sqrt{(2\pi)}}\int_{-\infty}^{\lambda} e^{-t^2/2}dt.$$

A channel responding to its "expected" signal as well as the noise has probability $\Phi(\lambda - s/n)$ of not exceeding λ.

The probability that the peak value will exceed λ in the absence of a signal is:

$$P(A|N)=1-\Phi^{M}(\lambda)$$

The probability that the peak value will exceed λ when the signal is presented is:

$$P(A|SN)=1-\Phi^{M-1}(\lambda)\Phi(\lambda-s/n).$$

These equations parameterize the ROC curve and have been plotted by Nolte and Jaarsma. The proportion correct in 2-alternative forced choice (2AFC) is the probability that the peak in the signal interval will exceed the peak in the other interval and is given by the area under the ROC curve (e.g. see Green and Swets 1966):

$$P(C)=\int_{\lambda=-\infty}^{\lambda=+\infty} P(A|SN)\ dP(A|N).$$

Substituting our expressions for P(A|N) and P(A|SN) yields:

$$P(C)= 1 - \int_{-\infty}^{+\infty} \Phi(\lambda-s/n)\ \Phi^{2M-2}(\lambda)\ \frac{d\Phi}{d\lambda}(\lambda)\ d\lambda.$$

---

[11] The Weibull function was a maximum likelihood fit to the data. The line was Nachmias and Sansbury's least-square error fit. A maximum likelihood fit of a Weibull function to a straight line with slope k (i.e. log d' $\propto$ k log c) has a β=k/.83.

Discussion

Figures 5.3a and 5.3b show the psychometric functions resulting from this equation.[12] The abscissa is the signal-to-noise ratio, s/n, and the ordinate is d'[13]. Figure 5.3a has log-log coordinates. Figure 5.3b has linear-linear coordinates. Proportion correct is also indicated, on a scale at the right. The functions are shown for values of M ranging from 1 to 10,000. Increasing M pushes the curve to greater s/n and greater steepness. The curves are nearly straight in log-log coordinates. The dashed curve is Peterson et al.'s approximation for M=100, and is obviously inaccurate for these values. Note that large changes in M, the number of expected signals, are required to produce a small change in slope. Nachmias and Sansbury's data in figure 5.2 would be well fit by a channel-uncertainty model with about 1000 expected signals. Their other observer had a shallower psychometric function (i.e. slope of 2.2) which would be well fit with about 50 expected signals. Tanner (1961) concluded that about 100 expected signals would account for the steep psychometric function for detection of a light increment. The large range of these estimates of the number of expected signals reflects the large

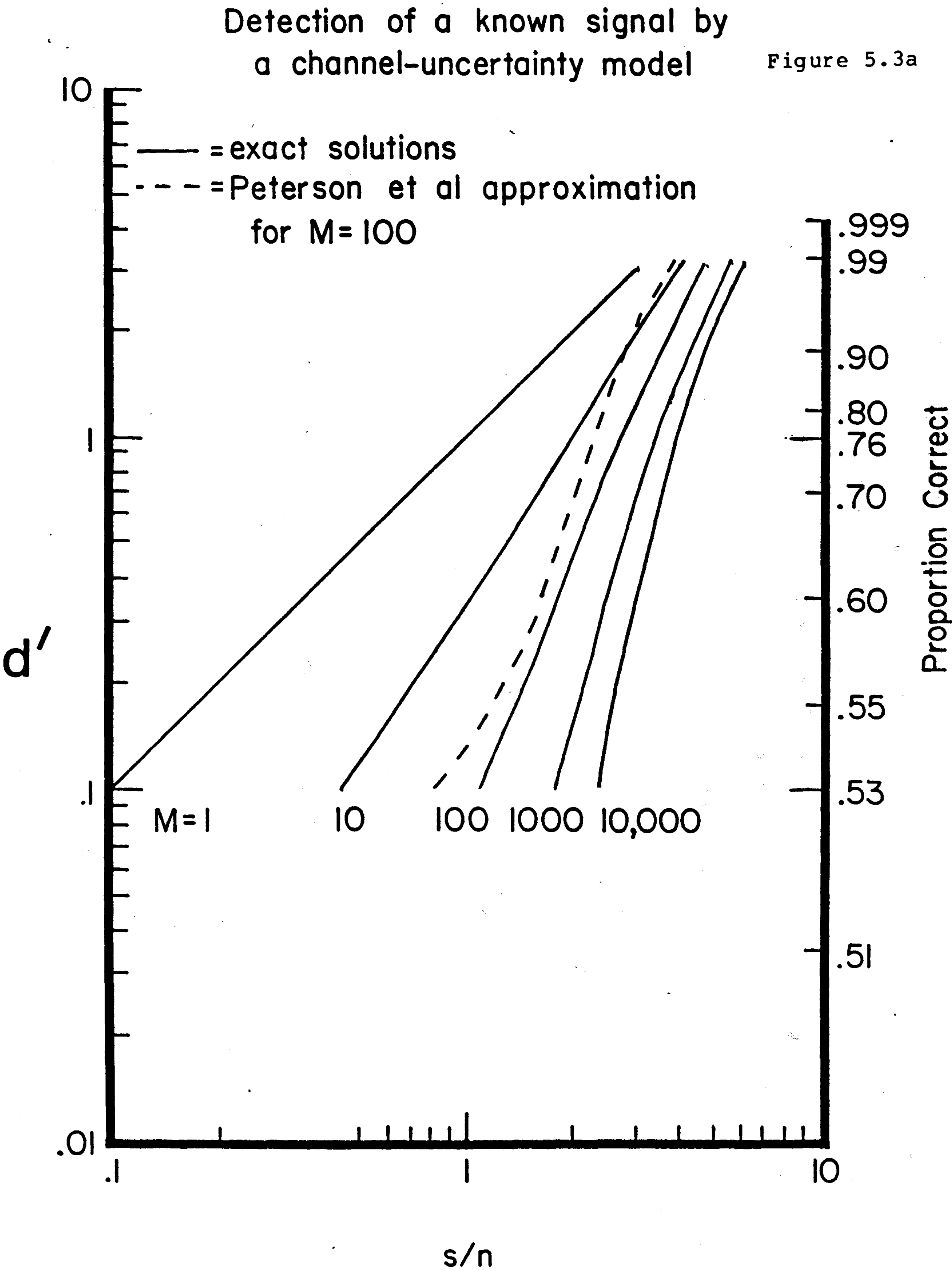
---

[12] The integral was evaluated by numerical integration. The cumulative normal function, $\Phi(\lambda)$, was approximated by a polynomial with an error of less than 7.5×10^{-8} (Abramowitz and Stegun, 1964, page 932, eq. 26.2.17). It was difficult to check accuracy because this calculation has never been made before. For the case M=1, d' did equal s/n, as it should. The d' values based on P(C) agree with Nolte and Jaarsma's d' values based on the negative diagonal intercept, but this is not a perfect test because these two values need agree only when the ROC curve has unity slope, which these do not.

---

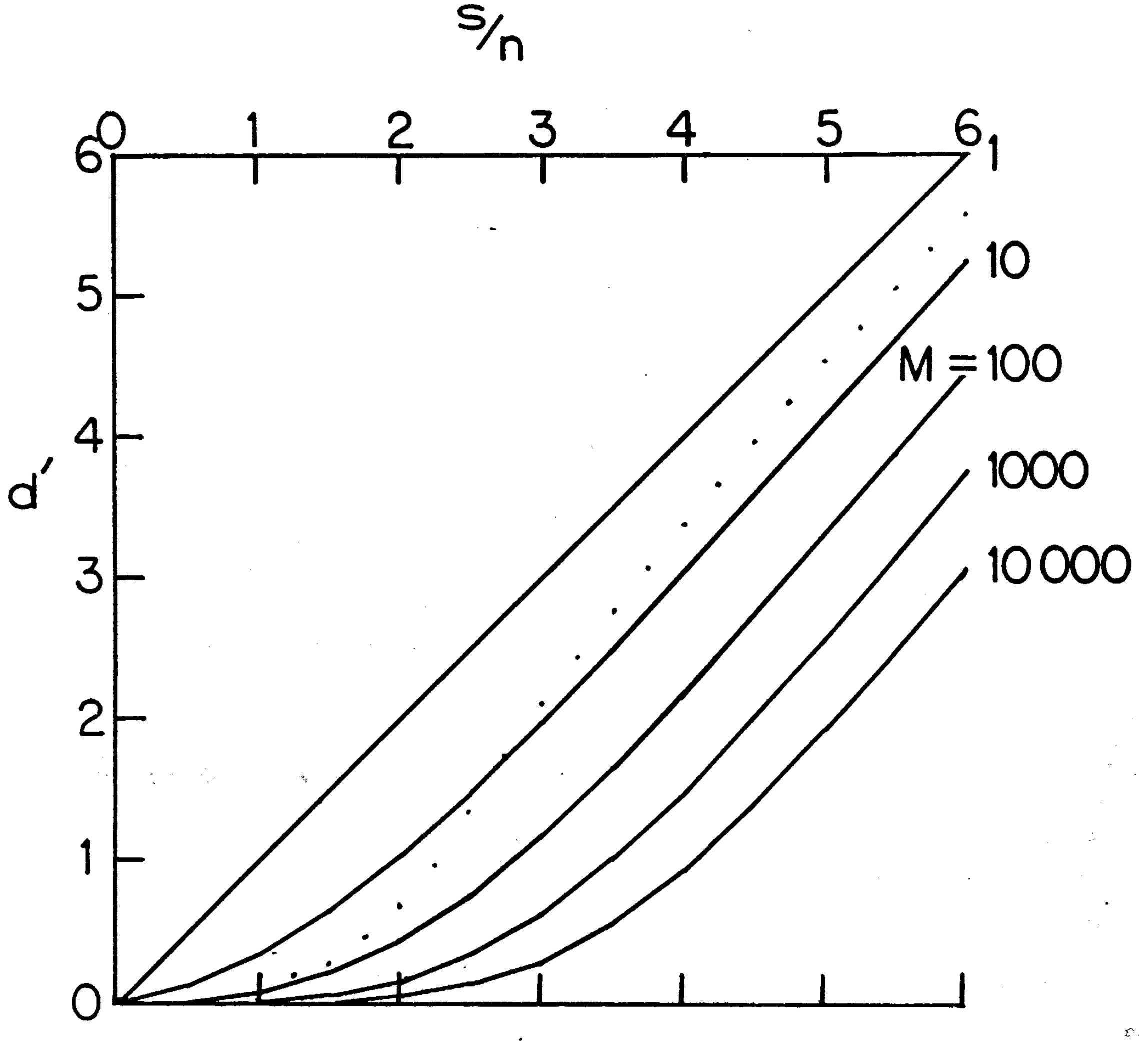
[13] d' is still calculated as $\sqrt{2}$ times the normal deviate corresponding to the proportion correct: the signal-to-noise ratio that an ideal observer (<u>that knew the signal exactly</u>) would require to equal the performance of the observer under study. This is because the experiment was such as to allow an ideal observer to obtain exact knowledge of the signal, but the uncertainty model, by assumption does not use this information. If the signal actually were one of M orthogonal equal-energy signals then d' would be nearly equal to the s/n, because the channel-uncertainty model is nearly ideal in that situation.

Figure 5.3a



Detection of a known signal by
a channel-uncertainty model
10
1
.1
.01
d′
——— = exact solutions
- - - = Peterson et al approximation
for M = 100
M = 1
10
100
1000
10,000
.1
1
10
s/n
.999
.99
.90
.80
.76
.70
.60
.55
.53
.51
Proportion Correct

Figure 5.3b



range of estimates of psychometric steepness and the fact that the model's steepness is proportional to the log of M.

These nearly linear curves suggest that we might characterize each curve by two numbers indicating its log-log slope, k, and signal-to-noise ratio, $(s/n)_{.76}$, at the point where d'=1 (and P(C)=76%). These two values determine a line in our double logarithmic coordinates:

$$d' = \left\{\frac{(s/n)}{(s/n)_{.76}}\right\}^k$$

$$\log d' = k \log\left\{\frac{(s/n)}{(s/n)_{.76}}\right\}$$

Figure 5.4a shows k versus M[14]. The slope, k, increases approximately in proportion to the logarithm of M. A second ordinate (on the right) shows β, the Weibull index of psychometric steepness, which is β=k/.83 (see the previous footnote). Figure 5.4b shows $(s/n)_{.76}$ as a function of M.

2. EFFECTS OF NOISE MASKING

2a. Human

Preceding chapters have presented evidence that squared threshold contrast is proportional to the effective noise level, which is the sum of the observer's equivalent noise and any actual luminance noise.

---

[14] The fitting procedure took advantage of the fact that the Weibull function is virtually straight in these coordinates with slope 0.83β (e.g. see figure 5.2). Proportions correct were calculated for the channel-uncertainty model. Then a maximum likelihood fit was made with the Weibull function to "dummy" data with 1000 "trials" at 20 signal-to-noise ratios and proportions correct as calculated for the model. A χ^2 statistic indicated that the fitted function was an excellent representation of the (dummy) data (Watson 1979). Finally the slope was taken as 0.83β.

Detection of a known signal by a channel-uncertainty model

Figure 5.4a

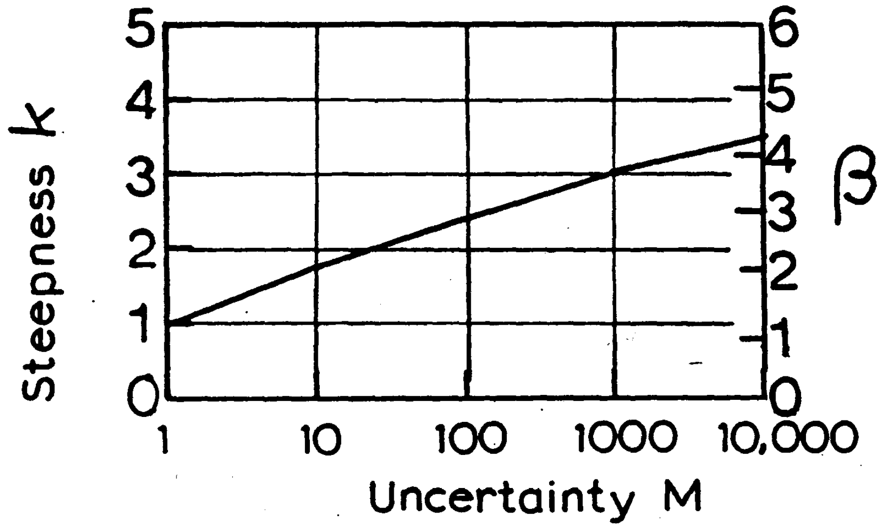


Figure 5.4b

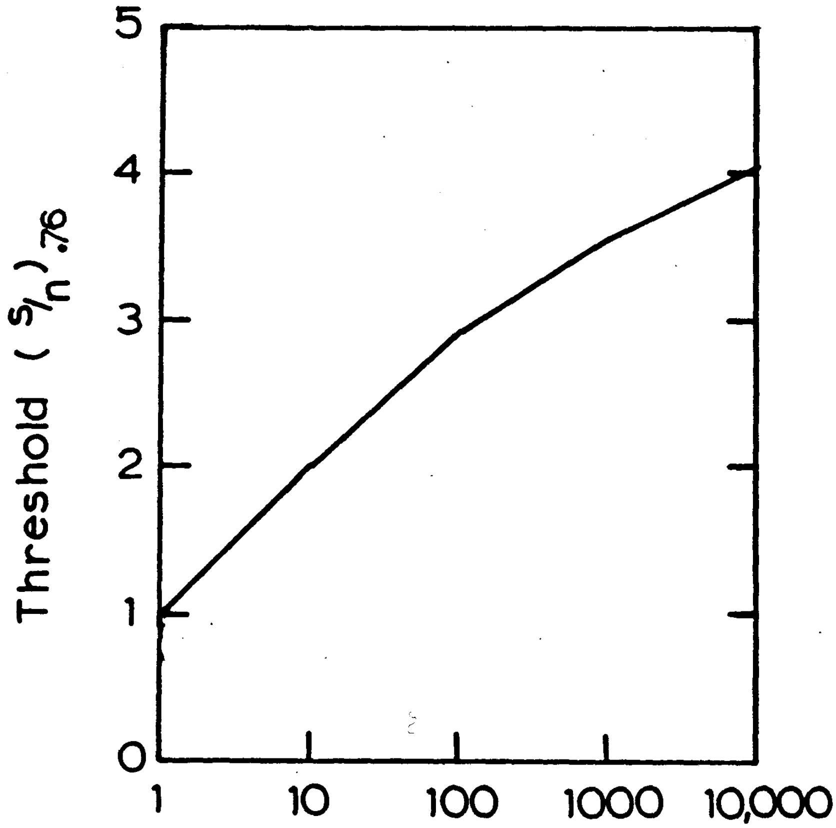
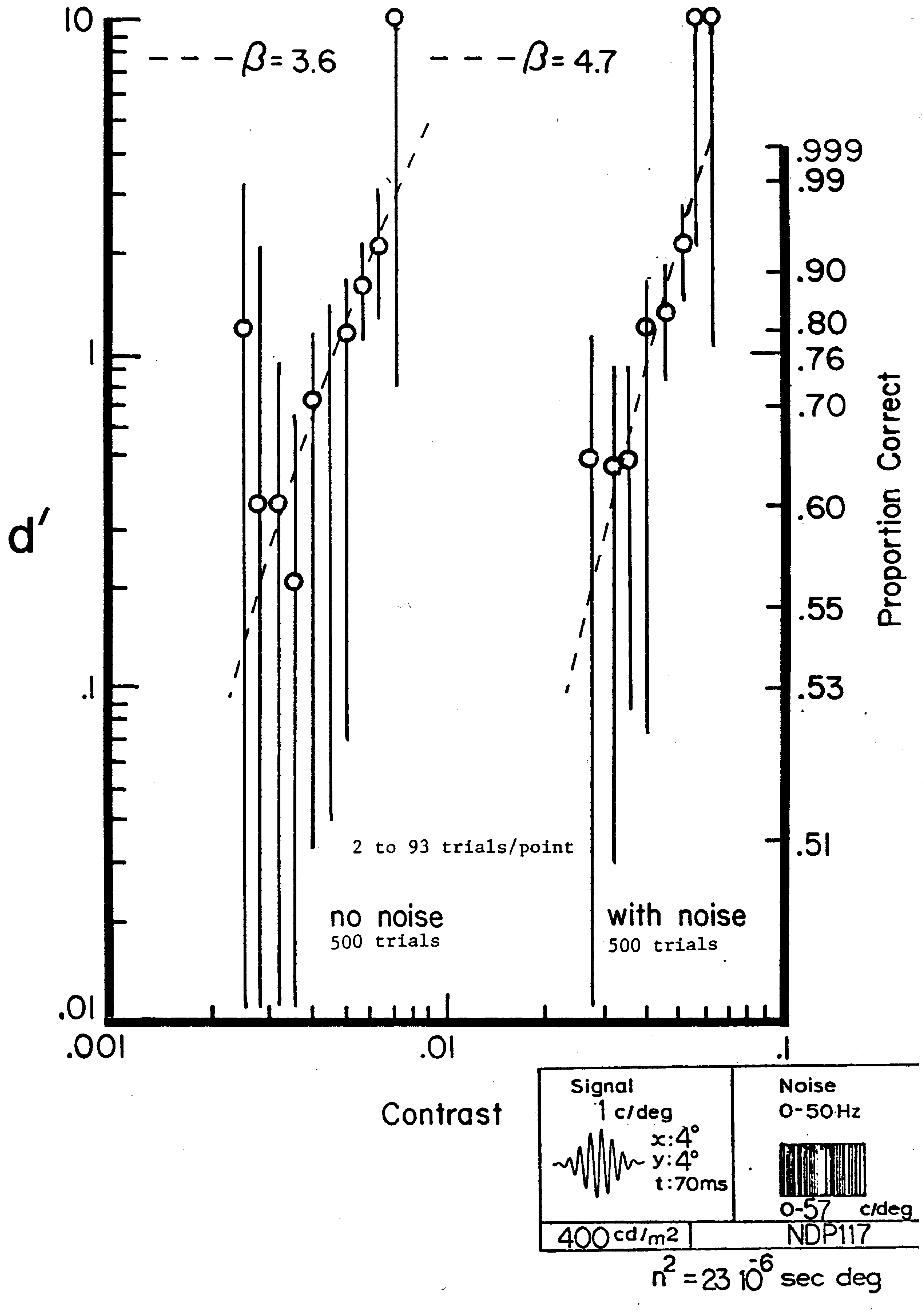


Figure 5.5 shows human psychometric functions for detection of a 1 c/deg grating without noise (on the left) and in the presence of dynamic, one-dimensional white noise (on the right). The solid curves are maximum likelihood fits (of the Weibull function). The vertical bars are 95% confidence intervals based on binomial statistics applied to the raw data at each contrast. The presence of the noise increased the contrast required for any level of performance by nearly an order of magnitude, but the shape of the psychometric function is hardly changed (in these coordinates). The in-noise curve on the right is slightly steeper, but not significantly so.

Griffiths and Nagaraja (1963) reported that their psychometric functions (proportion correct versus log contrast) had the same (steep) shape for detection of a disc with or without two-dimensional, dynamic, white noise. Cohn (1976) obtained steep psychometric functions ($d' \propto c^k$, $k>1$) for detection of a luminance increment with and without dynamic (zero-dimensional) noise. All reported psychometric functions in (continuously on) dynamic noise indicate a psychometric function steeper than $d' \propto c$.

However in static noise the situation is unclear. Henning, Hertz, and Hinton (1981) report steep psychometric functions for detection of gratings in static one-dimensional noise. Burgess, Wagner, Jennings, and Barlow (1981b) reported the same, but with static two-dimensional noise. Barlow (1978) and Burgess and Barlow (1981) report proportionality of d' to contrast for detection of large increments on a static dot display. This would indicate that (static) dot displays are the only exception, but Burgess (personal communication) finds that under some conditions he obtains unit slope with two-dimensional static noise. A consistent finding of a unit-slope psychometric function ($d' \propto c$) could be interpreted as evidence that the

Effect of noise on detection of 1c/deg Figure 5.5



observer's intrinsic uncertainty is purely temporal, and thus not detrimental when detection is limited by static noise. This interpretation is supported by the high 71% efficiency for detection recently reported by Burgess, Wagner, Jennings, and Barlow (1981a) for detection of a grating in static two-dimensional noise.

2b. Channel-uncertainty model

Noise added at the display just increases the effective noise level. The channel uncertainty model's performance is unaffected by proportional scaling of the squared contrast and the effective noise level. So increases in effective noise level will just shift the psychometric function to higher contrasts on our logarithmic contrast scale, without changing the function's shape.

This is an important point of agreement with the human data. Many other models of human detection assume stochastic processes that cannot be treated as an equivalent noise at the input, and thus predict very different behavior for detection with and without noise at the display.

2c. Probability summation

Nachmias (1981) has recently presented a clear analysis of the assumptions made in the standard form of probability summation calculations. In general, probability summation means only that - in a yes-no task - an observer will say "yes" if one or more of many independent events occurs. Nachmias points out that this includes the channel-uncertainty model. However, two other assumptions are usually made for analytical convenience, and these are not compatible with the channel-uncertainty model. According to the <u>high-threshold</u>

assumption there are no events in the absence of a signal, and according to the homogeneity assumption the probability of one or more events depends on signal contrast in the same way as the probability of any particular event does (except for a contrast-scale factor).

These events are usually described as a "detector" going into a "detect" state. Thus the high-threshold-version of probability summation incorporates noise only as the probability that each individual "detector" will go into a "detect" state in the presence of a signal. For convenience, every detector's probability of a "detect" is described by Weibull (1951) functions with the same steepness (homogeneity assumption) which is substantially steeper than the cumulative normal. By assumption the detectors never "detect" in the absence of a signal.[15] Presence of substantial effective noise at input would firstly increase every detector's probability of going into the "detect" state, even in the absence of a signal. One could suppose that an internal criterion was adjusted to reduce the "detect" rate to virtually zero in the absence of a signal, but the probability of any detector "detecting" would still have become a cumulative normal (which has a very low index of steepness). Thus noise at the input of a standard probability summation model would seriously change its behavior: violating its assumptions of high-threshold and homogeneity. This is because the high-threshold assumption is incompatible with the finding that

---

[15] This "high threshold" assumption makes the calculations simple but introduces serious theoretical problems (Nachmias 1981). To account for the fact that actual observers do occasionally say "yes" in the absence of a signal it is usually assumed that these "yes"s are due to an independent guessing process. However it has been well established that these "yes"s are not independent (Tanner and Swets 1954, Nachmias 1981) contradicting the high-threshold assumption. On this basis alone, the fact that the channel-uncertainty model does not incorporate the high-threshold assumption makes it a more plausible formulation of probability summation.

detection is unaffected by proportional increase of squared contrast and effective noise level.

2d. "Nonlinear transducer" model

Nachmias and Sansbury's (1974) "nonlinear transducer" model is not sufficiently explicit to make a prediction for the effects of noise at the visual field. They confined themselves to hypothesizing a nonlinear relation between an "internal effect" and signal contrast. The model amounts to the hypothesis that d' is a nonlinear function of contrast, which was known aleady, and what I call "d' additivity" (explained below), by which they accurately predicted contrast discrimination performance from the psychometric function for detection. They pointed out that channel-uncertainty would be one way of implementing their "nonlinear transducer" model.

Alternatively a more literal interpretation of "nonlinear transducer" would take the decision statistic to be a nonlinear transformation of signal contrast, to which a Gaussian internal noise is added. Foley and Legge (1981) have used such a model to accurately account for detection and discrimination data. There is no equivalent input noise for the noise in this model, and, by Birdsall's theorem (Lasely and Cohn, 1981), in the presence of a substantial noise at its input the internal noise and the nonlinear transformation would have no effect on performance, and the psychometric function should have unit slope (plotted as log d' versus log contrast), contrary to fact.

3. CONTRAST DISCRIMINATION: "d' ADDITIVITY"

3a. Human

We can discriminate contrasts that differ by less than the smallest contrast we can detect. This phenomenon is sometimes called "facilitation". Nachmias and Sansbury (1974) showed that facilitation may be related to the steepness of the psychometric function for detection. They showed that they could predict the d' for discriminating two contrasts by the difference in d's for detecting them. I call this finding d' additivity.[16]

Think of d' as a distance between two contrasts along a discriminability continuum. For detection one of the two contrasts is zero. Nachmias and Sansbury found that these distance add, so that the discriminability of c_1 from c_3 equals the sum of the discriminability of c_1 from c_2 plus the discriminability of c_2 from c_3.[17]

Foley and Legge (1981) have measured psychometric functions for detection and discrimination in order to test the d' additivity hypothesis. Their data were in excellent agreement with d' additivity. Can these data reject the channel-uncertainty model? The circles in figures 5.6a, 5.6b, and 5.6c represent proportions correct for observer JMF in detection of spatial

---

[16] The meaning of d' additivity depends on the definition of d'. I am sticking to the Tanner and Birdsall (1958) definition because it can be measured without assumptions about the observer. We will see that, by this definition, d' additivity is nearly, but not exactly a property of the channel-uncertainty model. Nachmias and Sansbury (1974) defined d' as the mean value of the decision statistic ("internal effect") on a signal and noise presentation minus the mean value on a noise-only presentation, all divided by the standard deviation on a noise-only presentation:

$$d' = (\mu_{SN}-\mu_N)/\sigma_N.$$

Nachmias (personal communication) has pointed out that by this definition of d', the channel-uncertainty model satisfies d' additivity exactly.

frequencies of .5, 2, and 8 c/deg. Each point represents about 200 to 300 2AFC trials. In Figure 5.6a error bars are shown for each point indicating the 95% confidence interval based solely on the binomial distribution. Foley and Legge (1981) determined the best values of M and scale factor (between s/n and c) to fit (by least squares) these data. That fit is shown by the continuous line running through the circles. As they noted it is a good fit. The squares represent proportions correct in contrast discrimination. They have been predicted by d' additivity (the dashed curve), and by the contrast discrimination performance of the same channel-uncertainty model used to fit the detection results (the solid curve, explained below). There were no degrees of freedom in either prediction. These data cannot reject either hypothesis. The extent to which the channel-uncertainty model fails to satisfy d' additivity may be too small to distinguish in a feasible experiment.

---

[17] To write this as an equation let $d'(c_1,c_2)$ represent the d' for discriminating contrasts c_1 and c_2. Specifically, they found that

$$d'(c_1,c_2)=d'(0,c_2)-d'(0,c_1)$$

so we may write

$$d'(0,c_1)=d'(0,c_3)+d'(c_1,c_3)$$

and

$$d'(0,c_2)=d'(0,c_3)+d'(c_2,c_3)$$

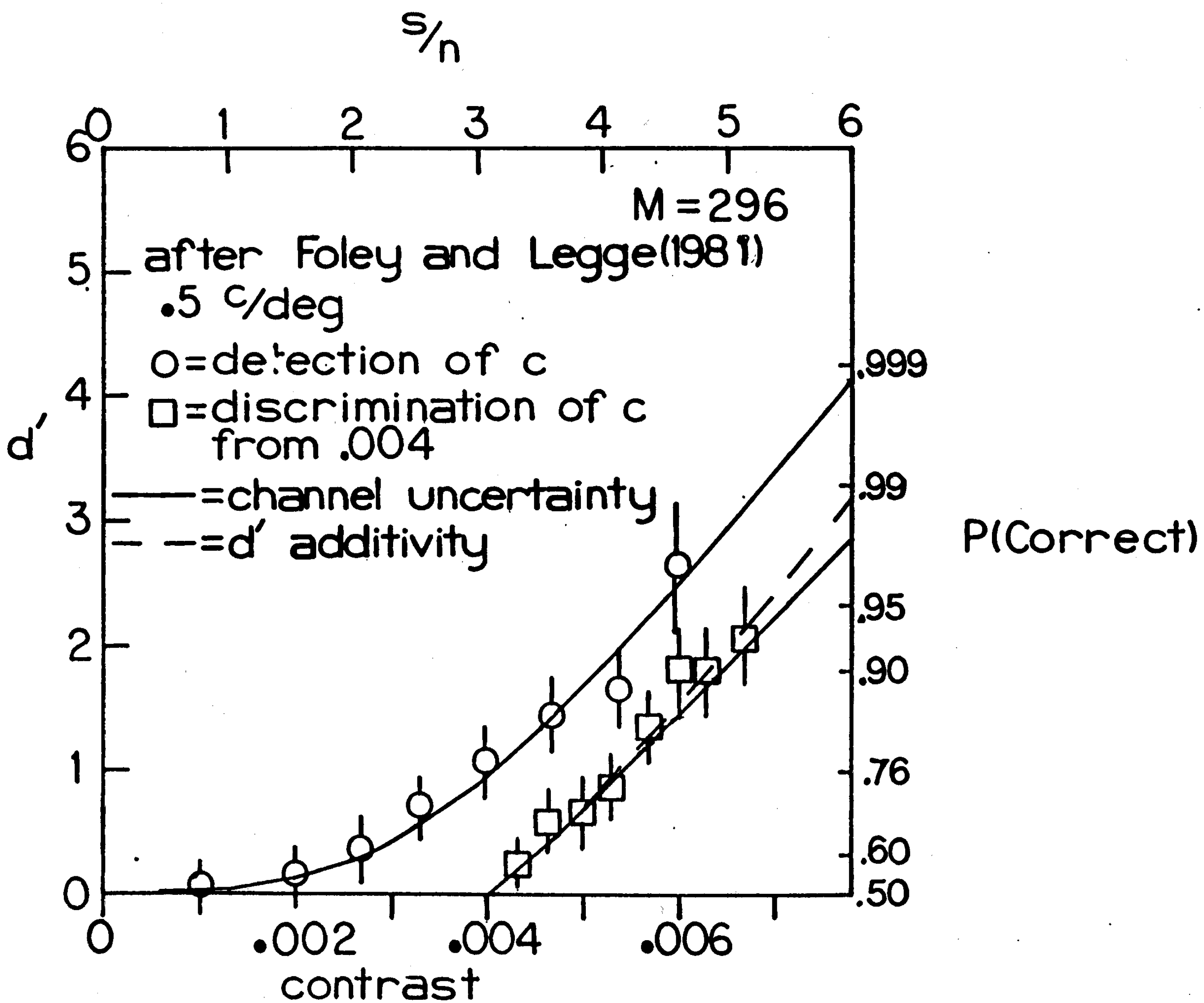
Substituting these quantities in the first equation gives

$$d'(c_1,c_2)=d'(c_1,c_3)-d'(c_2,c_3)$$

which is just the claimed d' additivity:

$$d'(c_1,c_3)=d'(c_1,c_2)+d'(c_2,c_3).$$

Nachmias and Sansbury examined values of c_1 ranging from 0 up to .0014, where $d'(0,c_1)\sim 2.4$, but they only used one value of c_2. However Foley and Legge (1981) have used several values of c_2 and confirmed d' additivity.



s/n
0
1
2
3
4
5
6
M = 296
after Foley and Legge (1981)
.5 c/deg
○ = detection of c
□ = discrimination of c from .004
—— = channel uncertainty
– – = d′ additivity
d′
0
1
2
3
4
5
6
P(Correct)
.999
.99
.95
.90
.76
.60
.50
0
.002
.004
.006
contrast

Figure 5.6a

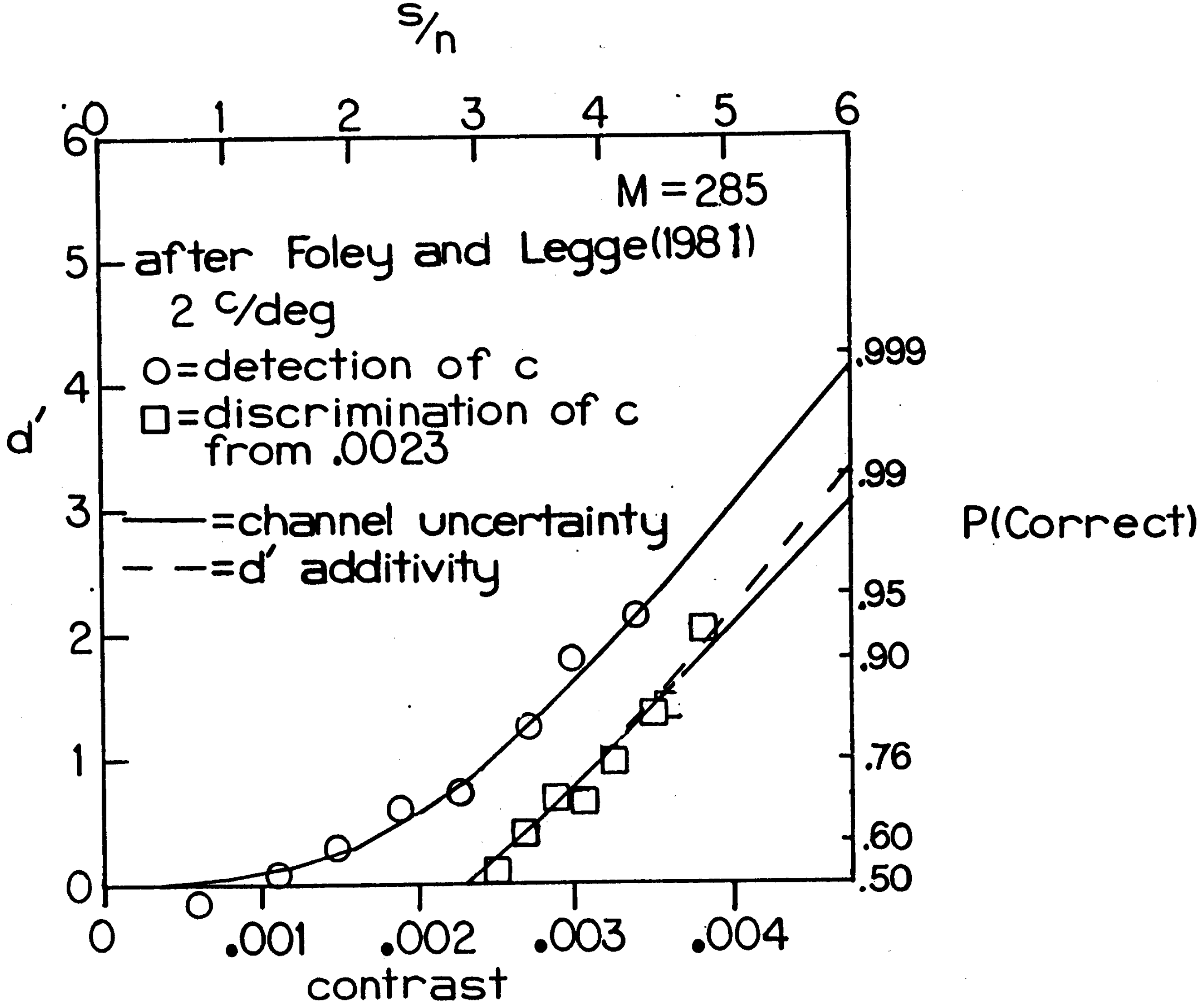
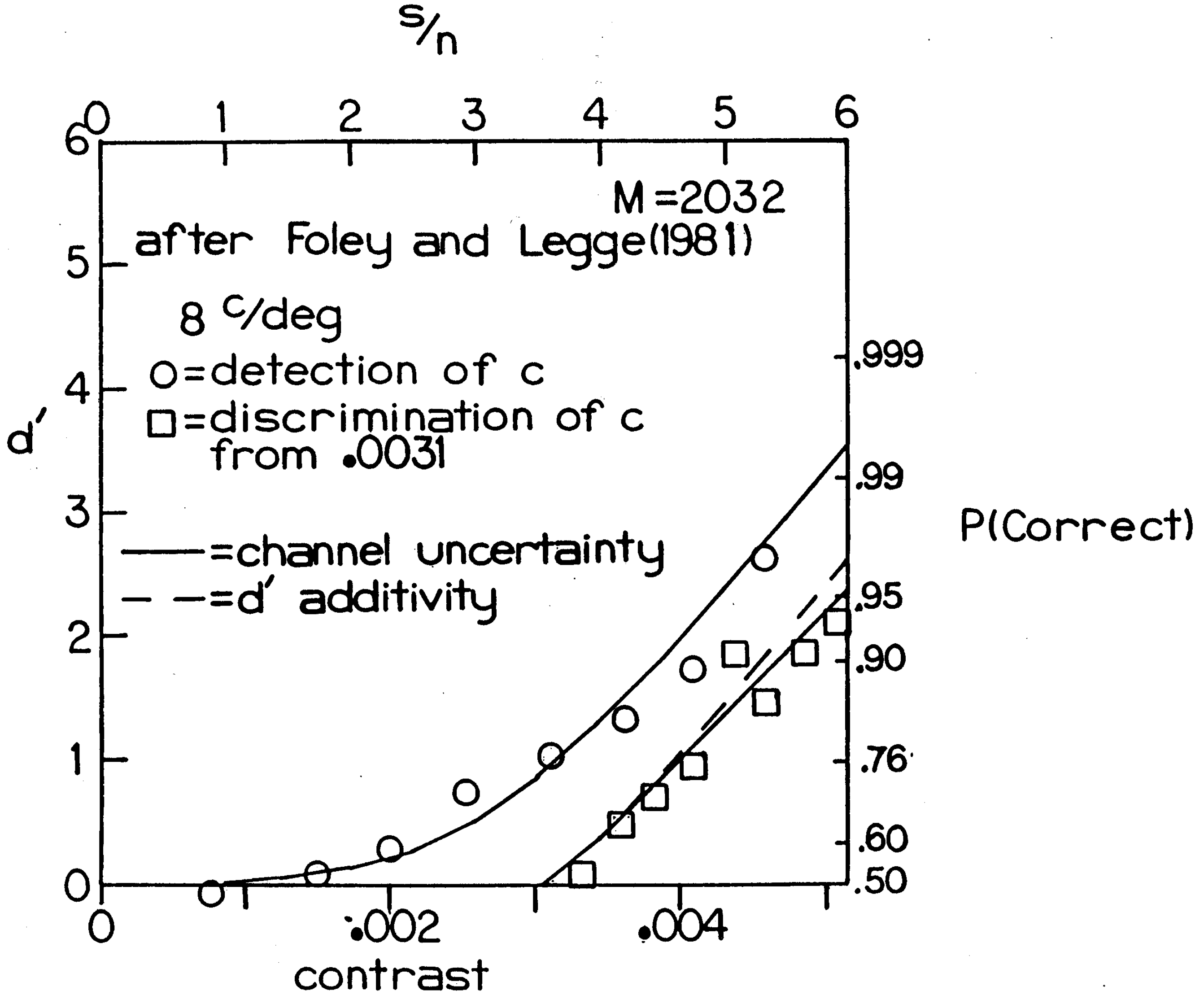

s/n
0 1 2 3 4 5 6
M = 285
after Foley and Legge (1981)
2 c/deg
O = detection of c
□ = discrimination of c from .0023
—— = channel uncertainty
– – = d′ additivity
d′
0 1 2 3 4 5 6
.999 .99 .95 .90 .76 .60 .50
P(Correct)
0 .001 .002 .003 .004
contrast

Figure 5.6b



s/n
0 1 2 3 4 5 6
M=2032
after Foley and Legge(1981)
8 c/deg
○=detection of c
□=discrimination of c from .0031
——=channel uncertainty
— —=d′ additivity
d′
0 1 2 3 4 5 6
P(Correct)
.999 .99 .95 .90 .76 .60 .50
0 .002 .004
contrast

Figure 5.6c

3b. Channel-uncertainty model

Theory

The contrast discrimination performance of the channel-uncertainty model may be calculated from our formula for $P(A|SN)$. Let the two signal strengths to be discriminated be s_1 and s_2. Then

$$P(A|S_1N)=1-\Phi(\lambda-s_1/n)\,\Phi^{M-1}(\lambda).$$

$$P(A|S_2N)=1-\Phi(\lambda-s_2/n)\,\Phi^{M-1}(\lambda).$$

These describe an ROC curve, and $P(C)$ is given by the area enclosed.

$$P(C)=\int_{\lambda=-\infty}^{\lambda=+\infty} P(A|S_1N)\; dP(A|S_2N)$$

$$=1-\int_{-\infty}^{+\infty} \Phi(\lambda-s_1/n)\;\Phi^{2M-3}(\lambda)\;[\Phi(\lambda)\frac{d\Phi}{d\lambda}(\lambda-s_2/n)+(M-1)\Phi(\lambda-s_2/n)\frac{d\Phi}{d\lambda}(\lambda)]\;d\lambda.$$

Discussion

Figures 5.6 and 5.7 plot d' as a function of the signal-to-noise ratio in the "signal" interval, (s_1/n), for several values of the signal-to-noise ratio in the "noise" interval, (s_2/n). As before d' is $\sqrt{2}$ times the normal deviate corresponding to $P(C)$. The uncertainty, M, is 1 in 5.7a, 10 in 5.7b, 100 in 5.7c, 1000 in 5.7d, and 10000 in 5.7e. The prediction of d' additivity is indicated by the dashed curves which are just vertical translations of the detection curve, ie.

$$d'(s_2,s_1)=d'(0,s_1)-d'(0,s_2).$$

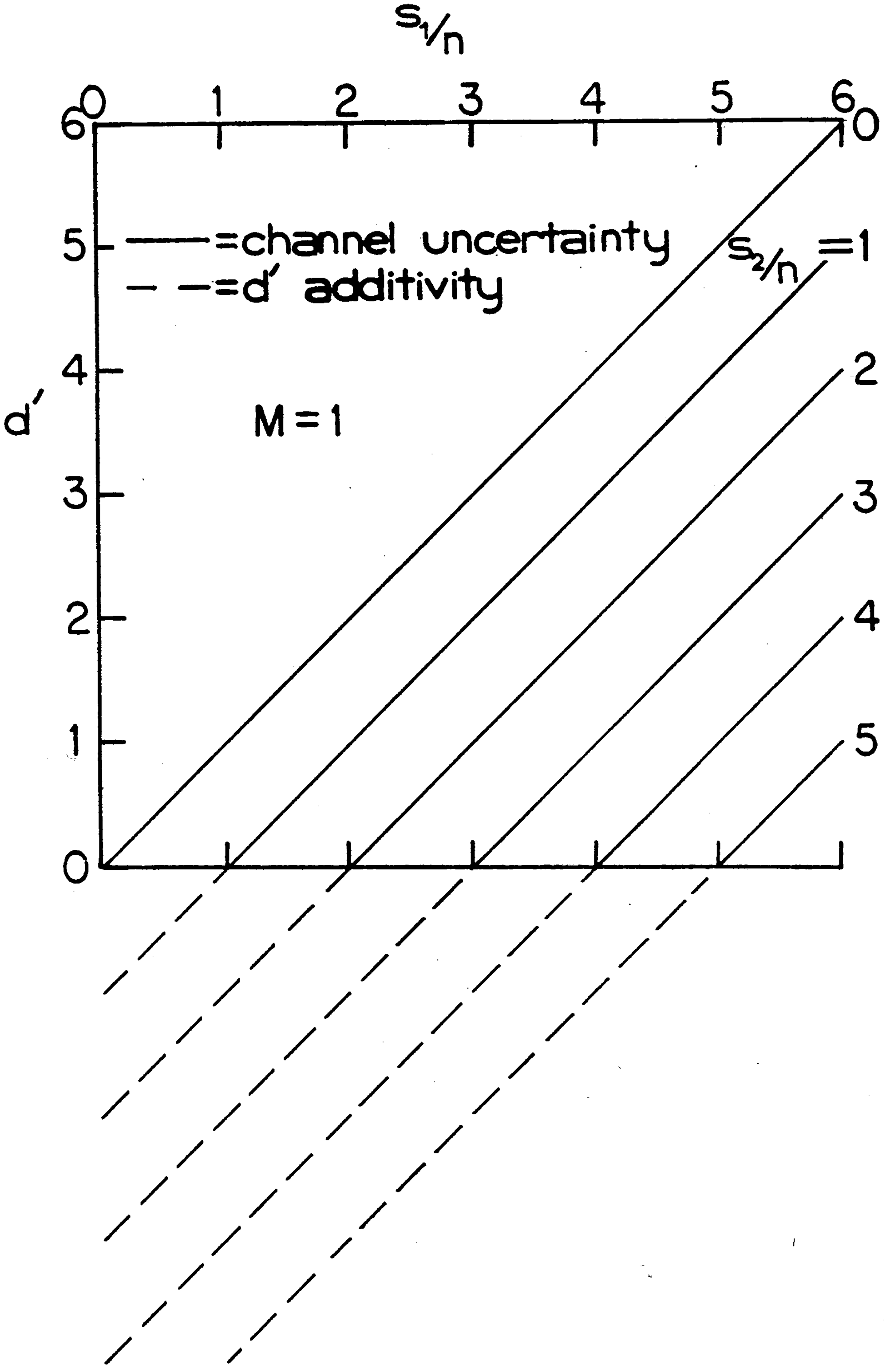
The deviations from d' additivity are quite small.

3'. CONTRAST DISCRIMINATION: FACILITATION

3'a. Human

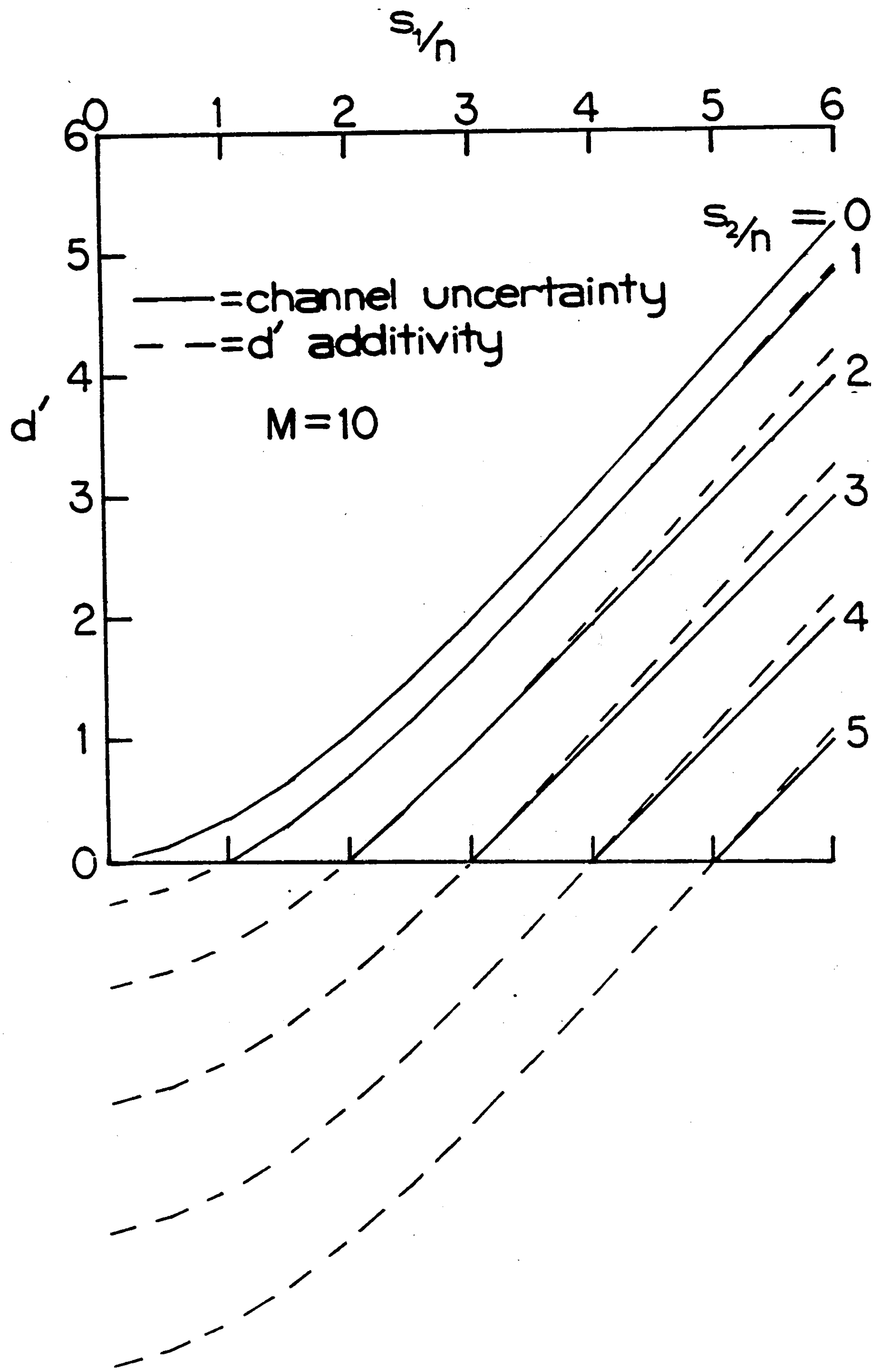
Thus far we have seen that the channel-uncertain model has two properties of human observers: firstly, its psychometric function for detection is well

Figure 5.7a



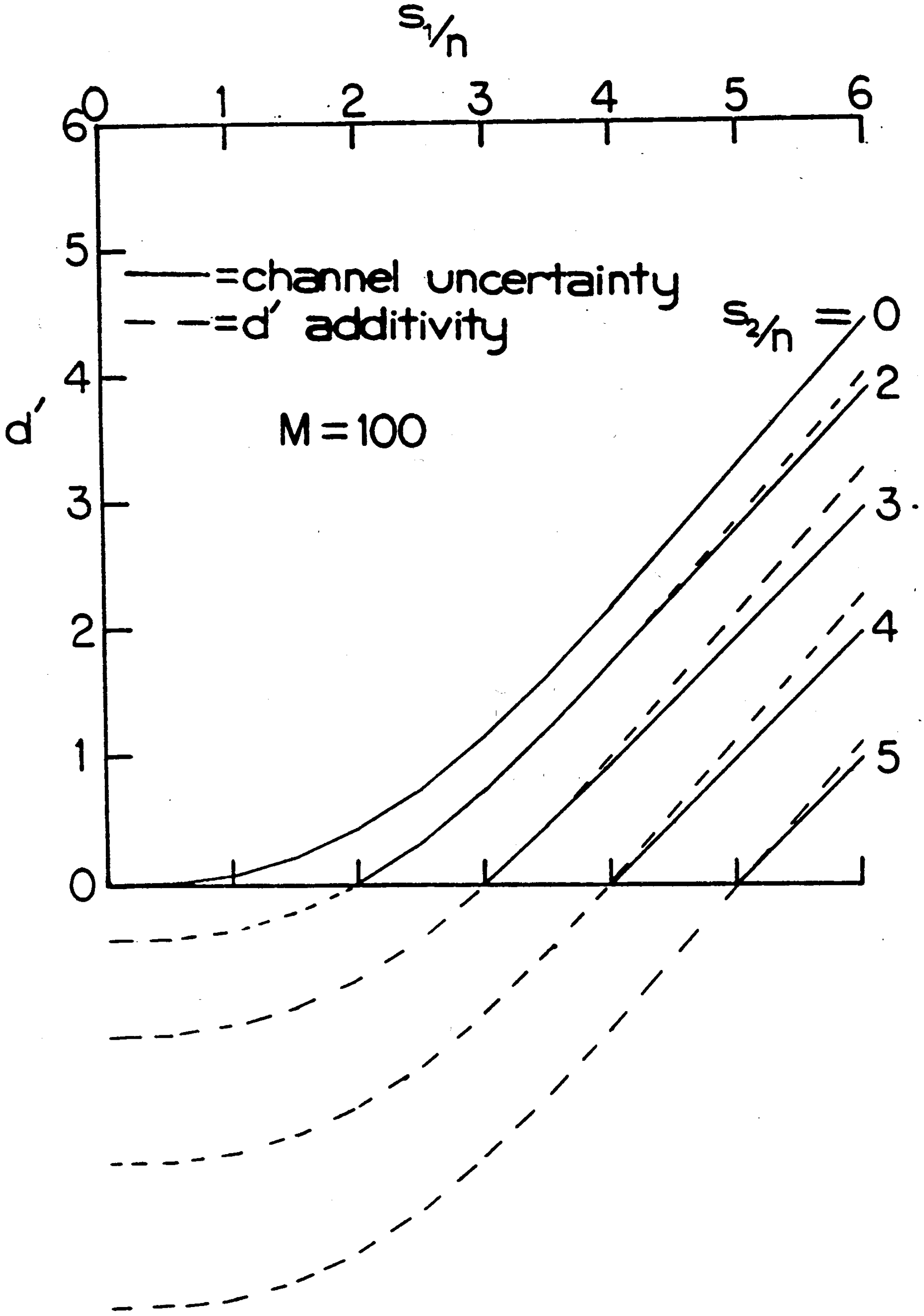
s_1/n
0
1
2
3
4
5
6
6
5
4
3
2
1
0
d′
—— =channel uncertainty
– – =d′ additivity
M = 1
s_2/n =1
0
2
3
4
5

Figure 5.7b

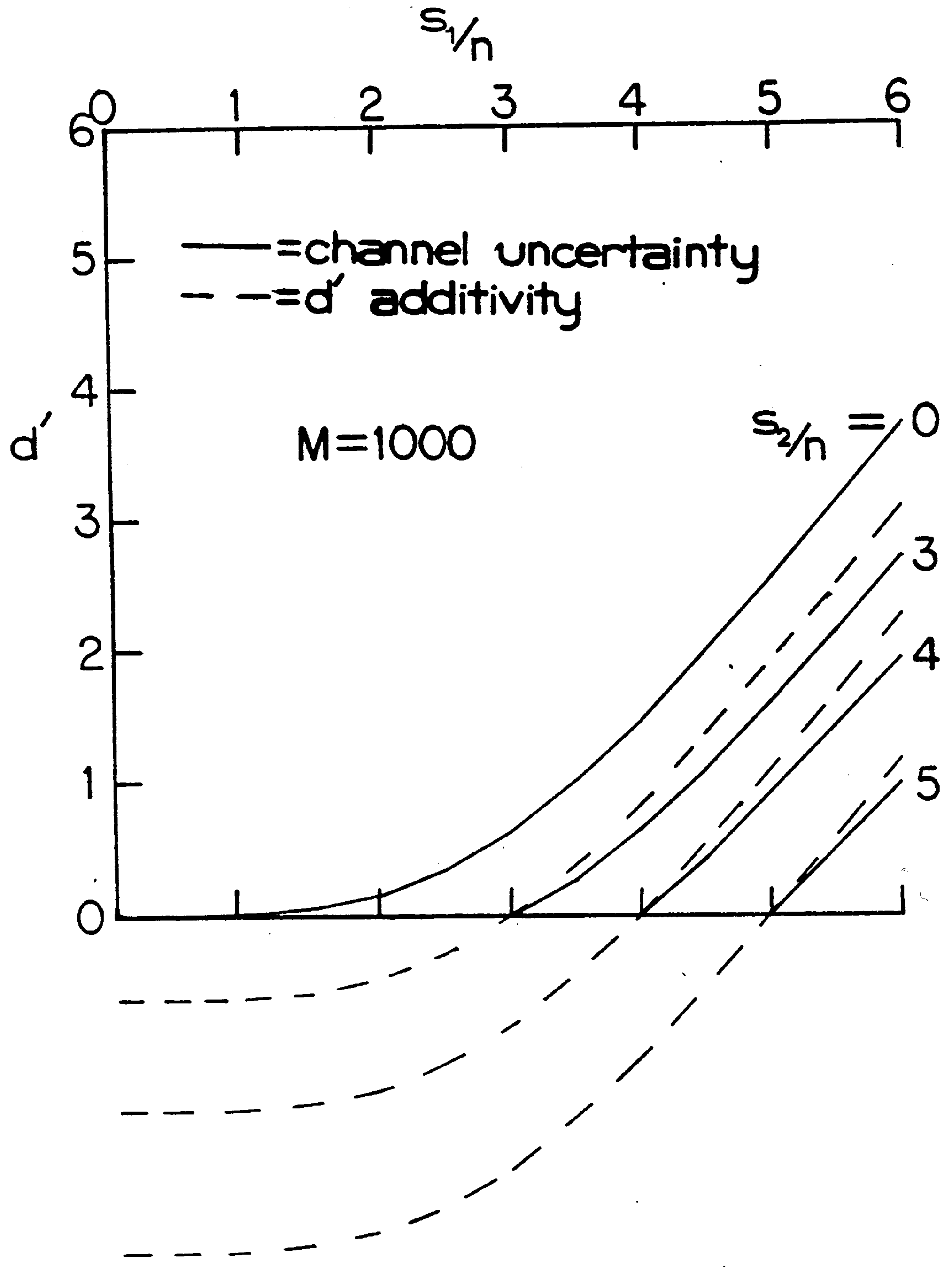


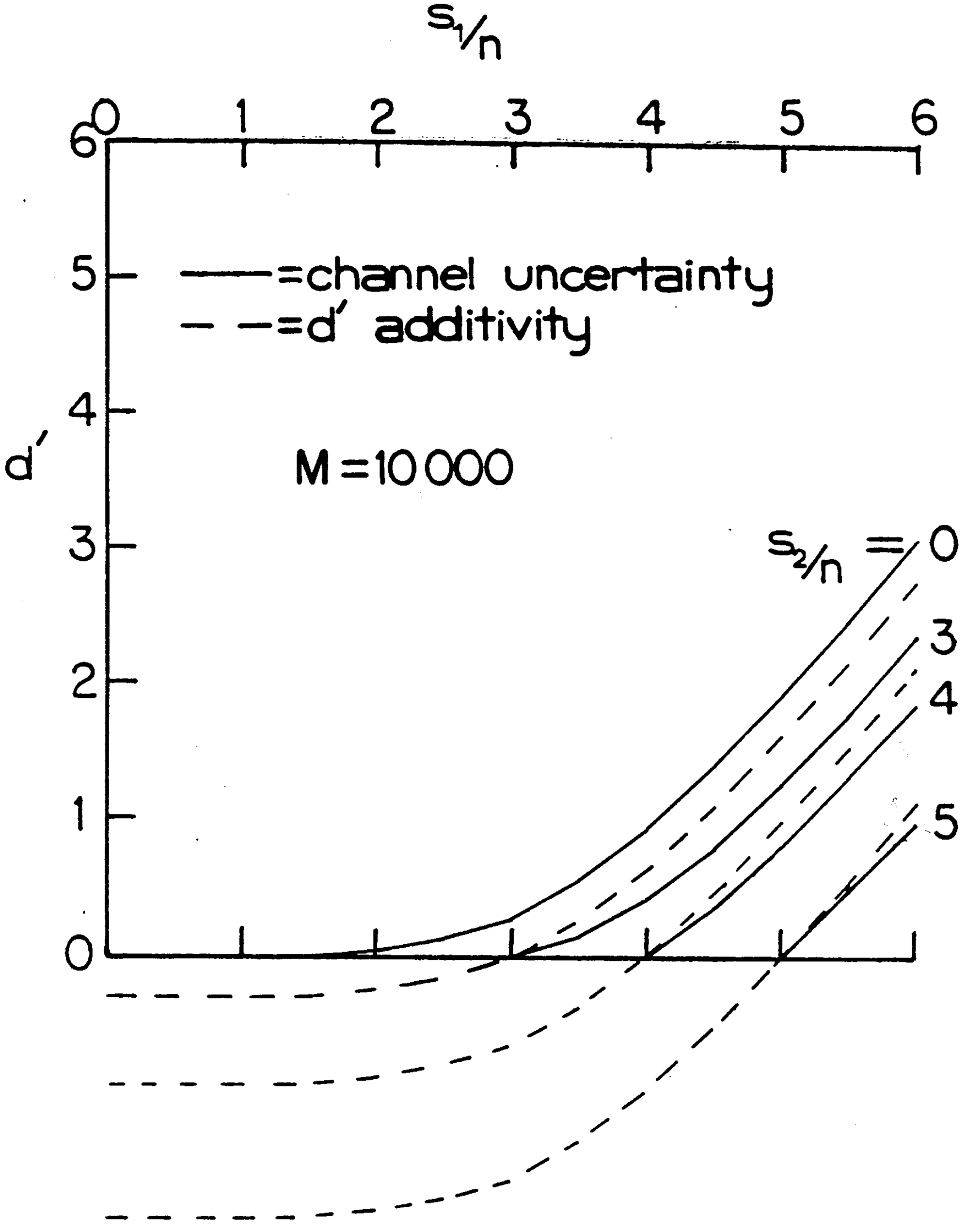
s_1/n
0 1 2 3 4 5 6
6 5 4 3 2 1 0
d′
——=channel uncertainty
– –=d′ additivity
M=10
s_2/n = 0
1
2
3
4
5

Figure 5.7c



s1/n
0
1
2
3
4
5
6
d'
0
1
2
3
4
5
6
—— =channel uncertainty
– – =d' additivity
M=100
s2/n = 0
2
3
4
5


s1/n
0 1 2 3 4 5 6
d′
0 1 2 3 4 5 6
——=channel uncertainty
– –=d′ additivity
M=1000
s2/n = 0
3
4
5



s_1/n
0 1 2 3 4 5 6
6 5 4 3 2 1 0
d'
—— =channel uncertainty
- - =d' additivity
M = 10 000
s_2/n = 0
3
4
5

approximated as $d' \propto c^k$, where k is greater than one (assuming M>1), and secondly, within experimental error, it obeys d' additivity.

Nachmias and Sansbury also reported the contrast difference threshold as a function of pedestal contrast. Threshold was taken to be the contrast difference required for some arbitrary percent correct. The contrast difference first fell with increasing pedestal contrast, and then rose. They explained that facilitation was related to the steepness of the psychometric function by d' additivity[18] (though they did not call it "d' additivity"). Following their argument we can predict the relation between the contrast difference threshold and the pedestal contrast, using just d' additivity and the steepness of the psychometric function for detection:[19]

$$\left(\frac{\Delta c_p}{c_p}\right)=\left\{\left(\frac{c}{c_p}\right)^k + 1\right\}^{1/k} - \left(\frac{c}{c_p}\right).$$

This equation has been graphed in figure 5.8 for various values of k. Note that the relation between $\left(\frac{\Delta c_p}{c_p}\right)$ and $\left(\frac{c}{c_p}\right)$ depends only on k. It is independent of the threshold criterion, p=P(C), so even method-of-adjustment thresholds could be fit by the relation to obtain an estimate of k.

Figure 5.9a (replotted from figure 4.1) shows $\left(\frac{\Delta c_{.82}}{c_{.82}}\right)$ and $\left(\frac{c}{c_{.82}}\right)$ for contrast discrimination in various noise levels: zero for the triangles, $.8\times10^{-6}$ sec deg^2 for the circles, and 8.4×10^{-6} sec deg^2 for the X's. Psychometric functions were not available to allow an exact prediction, but it may be seen that the relation is independent of the noise level. Since the near-threshold phenomenon scales with threshold contrast, and Chapter 2 showed

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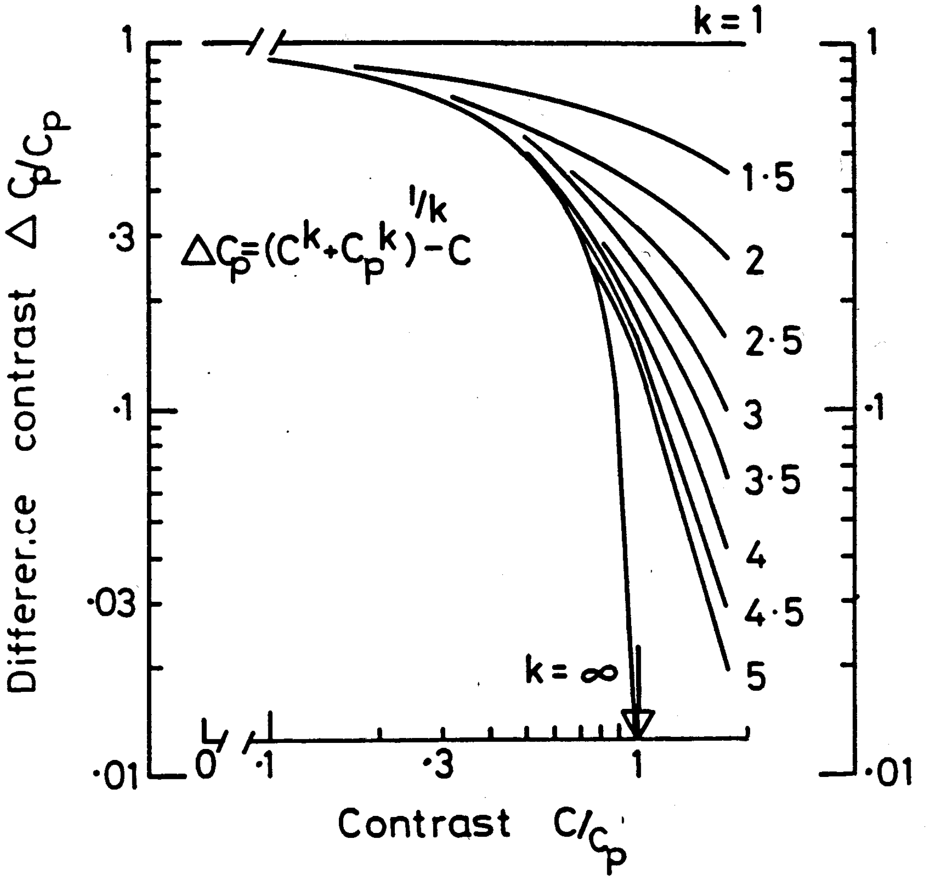
[18] Nachmias and Sansbury's (1974) different definition of d' does not affect how they calculated d' from human performance, only how they would calculate d' from models, so their statements about human performance do not need to be modified in order to be consistent with our usage.

Contrast discrimination as predicted by d′ additivity



The effect of noise on contrast discrimination.

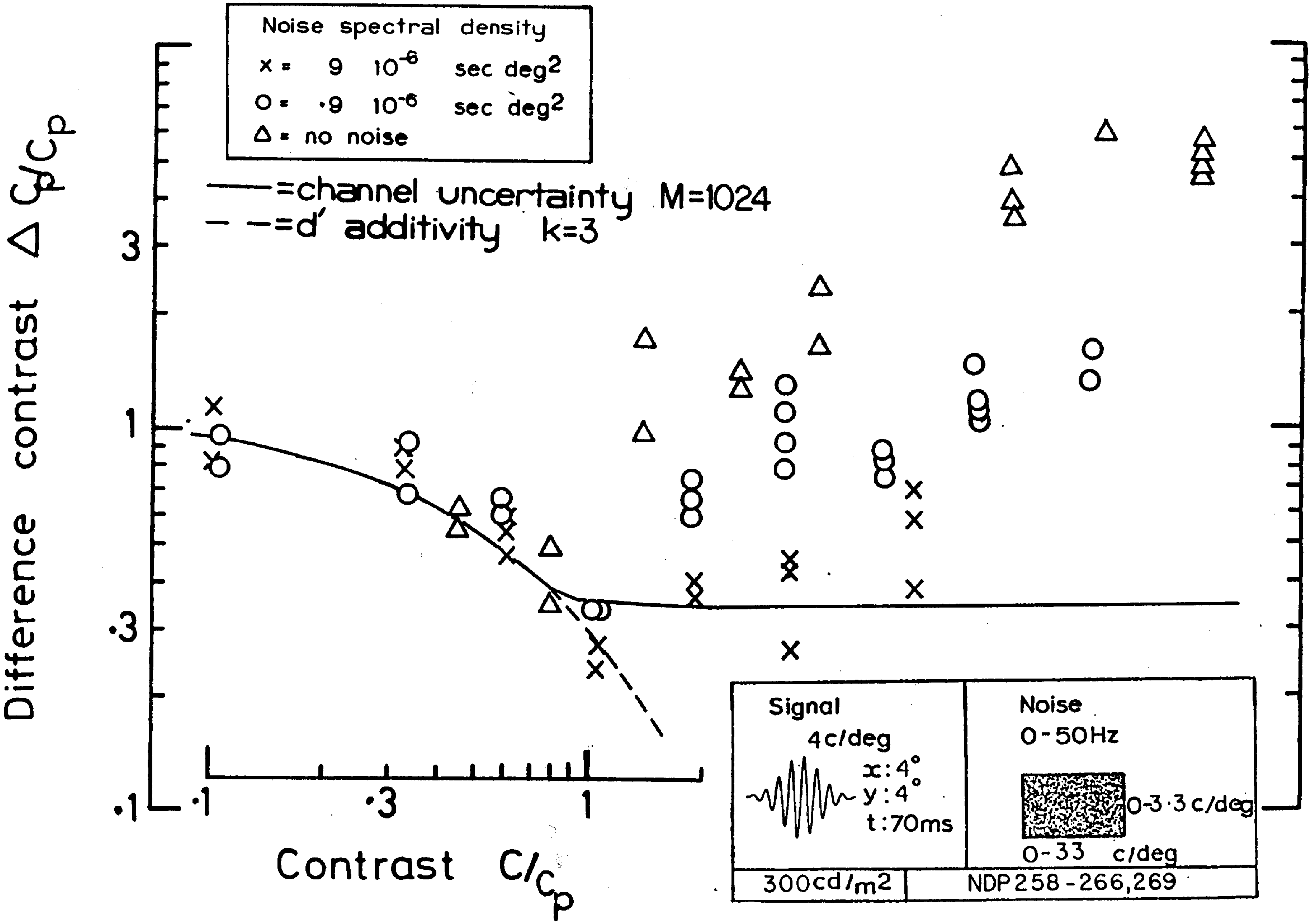


Figure 5.9a

that squared threshold is proportional to the effective noise level, this implies that near-threshold contrast discrimination is unaffected by proportional increase of squared contrast and effective noise level.

Note that all the data points for all noise levels are clustered along the falling curves (explained below) for subthreshold pedestal contrasts (i.e. $c/c_{.82} \leq 1$), and are widely scattered for suprathreshold pedestal contrasts (i.e. $c/c_{.82} > 1$). In a similar way the predictions coincide at

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[1] Let the two contrasts to be discriminated be c and $c+\Delta c$, where c is the "pedestal", and Δc is the contrast difference. d' additivity states that

$$d'(0,c+\Delta c)=d'(0,c)+d'(c,c+\Delta c).$$

Let Δc_p be the discrimination threshold, i.e. the contrast difference Δc for each pedestal c required for proportion correct p, i.e. $p=P(C)=\Phi(d'/\sqrt{2})$. And write $c_p=\Delta c_p$ for the detection threshold when the pedestal c is zero. Δc_p is defined so that $d'(c+\Delta c_p,c)$ is constant and independent of c, so

$$d'(c+\Delta c_p,c)=d'(0+\Delta c_p,0)$$
$$=d'(c_p,0)$$

Substituting into the d'-additivity equation we have

$$d'(0,c+\Delta c)=d'(0,c)+d'(0,c_p)$$

Now use the psychometric function $d' \propto c^k$ to substitute for d':

$$(c+\Delta c_p)^k=c^k+c_p^k,$$

Solving for Δc_p,

$$\Delta c_p=(c^k+c_p^k)^{1/k} - c$$

Dividing through by the detection threshold c_p leaves the desired result:

$$\left(\frac{\Delta c_p}{c_p}\right)=\left\{\left(\frac{c}{c_p}\right)^k + 1\right\}^{1/k} - \left(\frac{c}{c_p}\right).$$

When k is infinite the equation reduces to

$$\Delta c_p=\begin{cases} c_p-c \text{ for } c<c_p \\ 0 \text{ for } c>c_p. \end{cases}$$

which is a classical high threshold.

subthreshold pedestal contrasts, and diverge at suprathreshold contrasts (where no single curve could account for the data).

Recall that the performance of the channel-uncertainty model at contrast discrimination was calculated and plotted in figure 5.6. It is a simple matter to read off the contrast thresholds (e.g. where $d'=1$) for each pedestal strength. The solid curve in figure 5.9 is the contrast discrimination threshold for a channel-uncertainty model, where $M=1024$. As the pedestal is increased the discrimination threshold falls until the pedestal exceeds the detection threshold.

For comparison, the approximate slope, $k=3$, of the psychometric function was read off figure 5.4a for the 1024-channel observer. We have noted earlier that the psychometric function is approximately $d' \propto (s/n)^k$, and that d' additivity holds within experimental error. The dashed curve represents the prediction of d' additivity, assuming $k=3$. The two functions agree perfectly up to the threshold for detection. For larger pedestals the threshold for the channel-uncertainty model remains constant while the threshold based on d'-additivity and $d' \propto c^k$ continues to fall. This difference does not provide evidence against d' additivity. The psychometric function can only be measured for modest values of d' and the power law psychometric function may not hold beyond the measured range of d'.

The agreement among the data from different noise levels for subthreshold pedestals indicates that discrimination of subthreshold contrasts is unaffected by proportional change of (squared) contrast and noise level, and does not depend explicitly on absolute contrast. This is also the region of agreement with the predictions. This is reasonable: the d'-additivity prediction is valid over this region only, and the channel-uncertainty model, like the data, depends only on ratio of signal and noise, not on the absolute level of either.

The tremendous scatter of the points at suprathreshold contrasts shows that suprathreshold contrast discrimination does depend on absolute contrast. Figure 5.9b (a copy of 4.1) shows the original data, contrast discrimination threshold versus pedestal contrast. Note that the discrimination thresholds on the falling part of the curves (sub-threshold predestals) are all different at different noise levels, but are in excellent agreement over the rising part of the curve (supra-threshold pedestals), showing dependence on absolute contrast, and independence of noise level. This would seem to be evidence for signal-dependent noise, which is not a property of the channel-uncertainty model, though it could be incorporated easily without changing the prediction for subthreshold contrasts.

4. SUMMATION EFFECTS

4a. Human

Over the past few years there has been some importance attached to studies of summation, that is, measurements of the contrast threshold for sinusoidal gratings as a function of number of bars (e.g. Legge 1978b, Robson and Graham 1981), or exposure duration (e.g. Legge 1978, Watson 1979). Over a wide range in each dimension the contrast threshold falls slightly if the grating is extended in space or time. Over this range threshold is inversely proportional to the extent raised to a small exponent, usually called the index of summation. Using the standard probability-summation calculations these studies and others were able to successfully predict that the index of summation was equal to $1/\beta$, the reciprocal of the index β of psychometric steepness.

The effect of noise on contrast discrimination.

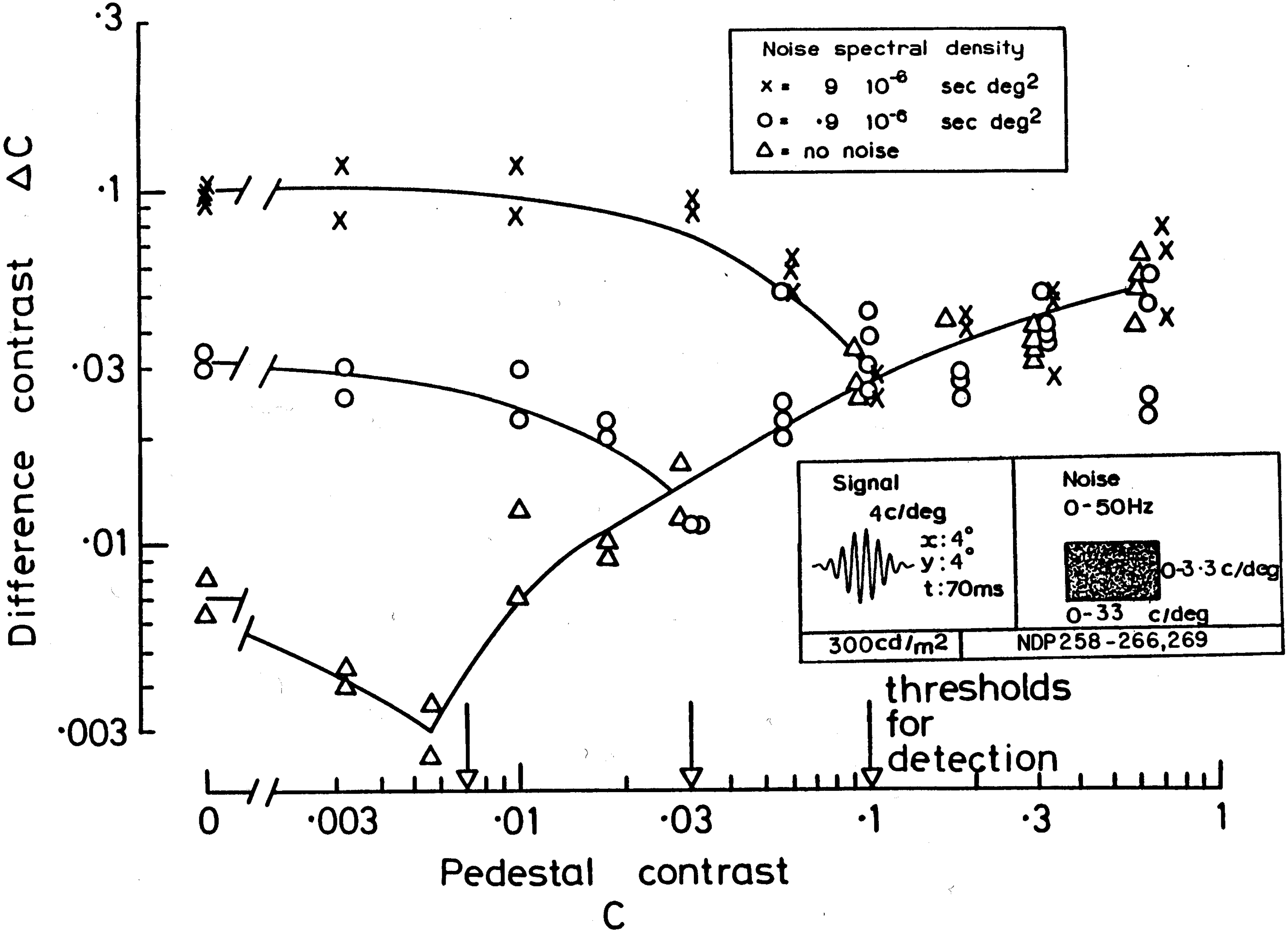


Figure 4.1 and 5.9b

Figure 5.10 shows a psychometric function from Legge (1978) for detection of a 180 msec 3 c/deg grating. It is well fit by a Weibull function,

$$P(c) = 1-e^{\alpha c^{\beta}},$$

with exponent $\beta=2.44$, shown by the dashed curve. The solid curve, also a good fit, represents a 32-channel uncertainty model.

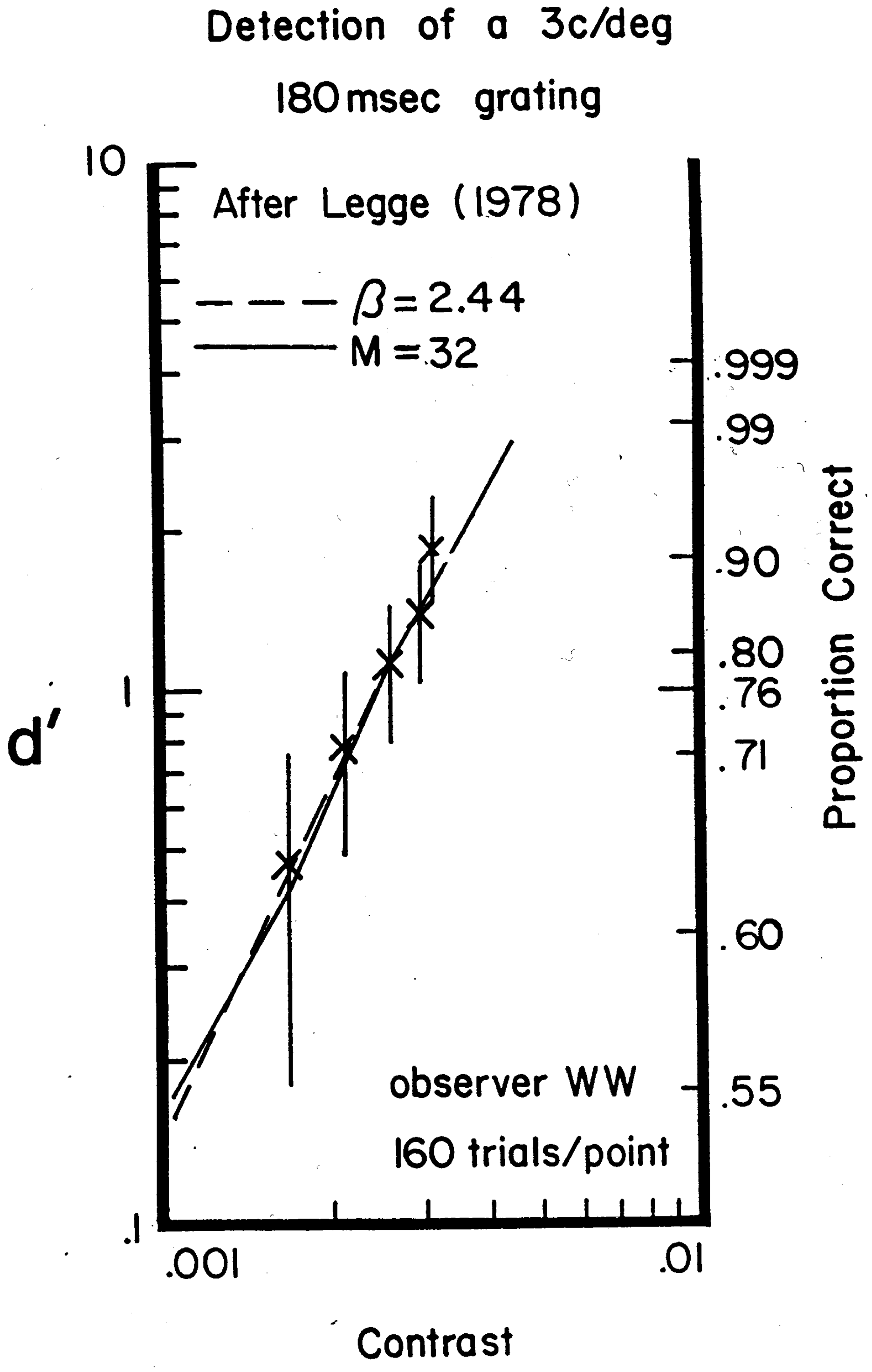
Figure 5.11 shows the thresholds reported by Legge for durations 180 to 3000 msec. He took threshold to be 71% correct. The dashed line shows the prediction of probability summation, based on the psychometric function we just saw for the 180 msec duration. That is,

$$c \propto \text{duration}^{-1/\beta}.$$

The solid lines connect the predictions of the channel-uncertainty model assuming the signal stimulated from 1 to 16 of the total of 32 channels (explained below). Note that up to 1000 msec the predictions are both reasonably close to the data, though the predictions are somewhat steeper than the data. The discrepancy is not significant because there is a large variance associated with estimating the index of steepness of the psychometric function. The important point is the great similarity in the predictions of these two forms of probability summation. No one has reported this fact before.

4b. Channel-uncertainty model

The calculations up to now assumed that only one channel was stimulated by the signal. Here it will be supposed that as a grating is extended it stimulates more and more of the channels. Since M was derived from the psychometric function for the 180 msec condition, it will be assumed that only



Detection of a 3c/deg
180 msec grating
After Legge (1978)
β = 2.44
M = 32
observer WW
160 trials/point
10
1
.1
d′
.001
.01
Contrast
.999
.99
.90
.80
.76
.71
.60
.55
Proportion Correct

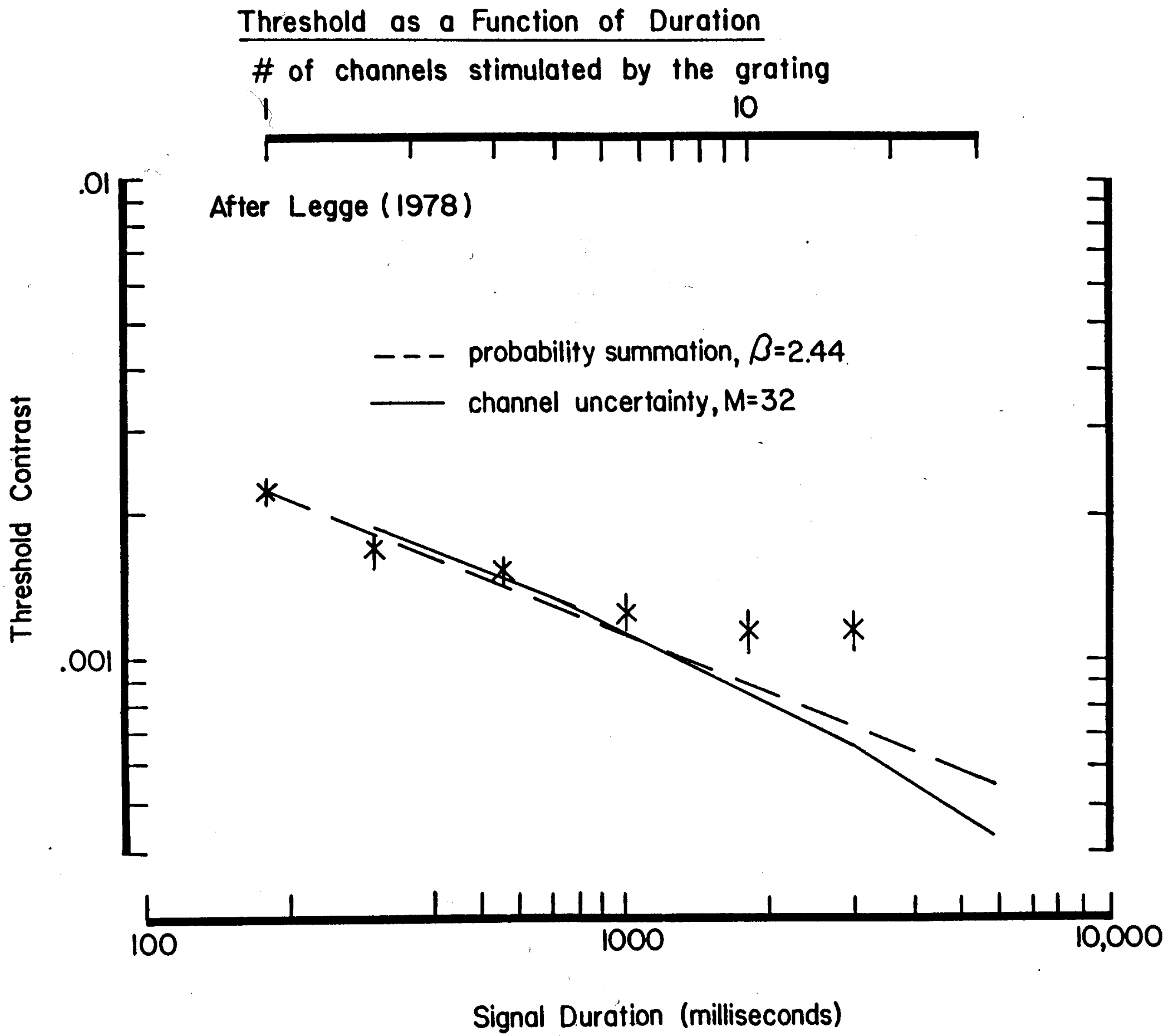

Threshold as a Function of Duration
# of channels stimulated by the grating
1
10
.01
.001
After Legge (1978)
probability summation, β=2.44
channel uncertainty, M=32
Threshold Contrast
100
1000
10,000
Signal Duration (milliseconds)

Figure 5.11

one channel bears the signal in that condition, and that for longer signal durations a proportionate number of channels will bear parts of the signal.

Theory

We have simultaneous presentation of a signal with N parts, each with signal-to-noise ratio $\frac{s/n}{\sqrt{N}}$ (so the total signal-to-noise ratio is s/n, as always). We have

$$P(A|SN)=1 - \Phi^{M-N}(\lambda)\ \Phi^{N}(\lambda - \frac{s/n}{\sqrt{N}})$$

$$P(A|N)=1 - \Phi^{M}(\lambda).$$

As before

$$P(C)=\int_{\lambda=-\infty}^{\lambda=+\infty} P(A|SN)\ dP(A|N).$$

Substituting in P(A|SN) and P(A|N) gives

$$P(C)=1 - M \int_{-\infty}^{+\infty} \Phi^{2M-N-1}(\lambda)\ \Phi^{N}(\lambda - \frac{s/n}{\sqrt{N}})\ \frac{d\Phi}{d\lambda}(\lambda)\ d\lambda.$$

Discussion

Figure 5.11 shows that the prediction of the channel-uncertainty model is very similar to that of the standard version of probability summation. To test the generality of this conclusion similar calculations were made for 1024 channels. The psychometric function of this model is well fit by a Weibull function with $\beta=4.0$ (as shown by figure 5.2). The predicted thresholds are virtually identical for 1 to 64 signal-bearing channels, and differ by less than a factor of $\sqrt{2}$ up to 512 signal-bearing channels. Thus the summation predictions by the high-threshold version of probability summation and by the channel-uncertainty model do not differ appreciably. The similar predictions by the two models should not be surprising; they are both probability summation models.

We have seen that the two models are readily distinguished by adding noise at the input, which defeats the high-threshold assumption. In principle, summation experiments too could distinguish the two models, but the difference in the predictions may be too small to measure.[20]

[20] The standard version of probability summation explicitly assumes the psychometric function steepness to be constant (Nachmias, 1981, calls this the "homogeneity" assumption), while as more and more of the channel-uncertainty model's channels bear signals its psychometric function becomes more and more shallow, reaching $d' \propto c$ when all its channels bear signals. However my calculations of this effect show that a large proportion of the channels must be stimulated by the signal before β changes much. It would be difficult to obtain a large enough range of signal extent and sufficiently precise estimates of β in order to distinguish the slightly different predictions of the two forms of probability summation. Uncertainty among 1024 channels produces a β of 3.5 for a one-channel signal. 64 channels must be stimulated for β to drop to 2.5. 200 channels must be stimulated for β to drop to 2. Summation experiments have covered ranges of signal extent much less than this, and estimates of β are notoriously variable. For example, Watson (1979) varied signal duration from 100 msec to 800 msec, only an 8:1 range, and reported 52 estimates of β for observer RP with a mean of 4.7 and a standard deviation of 1.4. None of the summation studies report the steepness of the psychometric function as a function of signal extent.

TESTING THE UNCERTAINTY HYPOTHESIS

The uncertainty hypothesis predicts that since the observer makes little or no use of prior information about signal identity he should perform nearly as well without it. Appendix 6 presents the results of an experiment which measured the effect of depriving the observer of the prior information about signal identity which is usually made available to him. There have been several experiments which compared detection of one stimulus with detection of one of several of disparate stimuli, and found that the experimental uncertainty caused a small reduction in detectability. For example Cohn and Lasley (1974) compared detection of a spot at one, or at one of four disparate locations; Greenhouse and Cohn (1978) compared detection of a spot of one, or one of several colors; Davis and Graham (1979) compared detection of a grating of one, or one of several spatial frequencies. At first blush these experiments would seem to be evidence <u>against</u> the uncertainty hypothesis because they indicate that introducing only a small uncertainty reduces the observer's performance measureably, implying that the observer <u>was</u> making use of the information available in the certainty condition. However, asking an observer to detect one of several disparate signals may result in an enormous increase in his uncertainty if he monitors the entire area, hue range, or spatial-frequency bandwidth that the set of possibilities extend over.

The experiment reported in Appendix 6 compared detection of a thin $(\frac{1}{4}^{\circ})$, brief (20 msec), bright line at a fixed time and location, with detection of the same at one of ten-thousand possible <u>contiguous</u> times and places. Thus the uncertainty condition included all possible, nonoverlapping, signals within an area 24° wide and a duration 2000 msec long. The threshold in the uncertainty condition was only a factor of 1.5 higher than in the certainty condition. It is not necessary to believe that the observer can resolve all ten-thousand signals; even if he could only resolve signals separated by 100 msec, the uncertainty was 2000. Thus, as predicted by the uncertainty hypothesis, drastic reduction in the prior information had little effect on observer performance.

CONCLUSIONS

The channel-uncertainty model behaves as humans do in at least four respects:

1. The psychometric function is steep ($d' \propto c^k$, $k>1$).

2. Detection and discrimination of near-threshold contrasts are unaffected by proportional increase of squared contrast and effective noise level.

3. d' is additive within experimental error, that is, discriminability of two contrasts equals the difference in their detectabilities, so the steepness of the psychometric function for detection implies the existence of facilitation in discrimination.

4. Summation effects: the small index of summation can be predicted from the high index of steepness of the psychometric function.

Various theories of visual detection embody idealizations of relations observed empirically. Over the years study of detection and discrimination has yielded several simple relations which are analytically convenient and which give good account of some aspect of detection. These relation accurately predict summation effects (by probability summation) and contrast discriminability (by d' additivity) from the psychometric function for detection. To these relations we may add the fact that detection and near-threshold discrimination are unaffected by proportional increase of squared contrast and effective noise level. Past suggestions that uncertainty may underlie the steepness of the psychometric function and its relation to discrimination, have been extended here to show that the channel-uncertainty model exhibits the observed relations (summation effects, d' additivity[21], and independence of c^2/n^2) within experimental accuracy, though not with the analytic convenience of models which each accounted for only one of the phenomena. Within their intended realms of application the more restricted models do not seem to be experimentally discriminable from the channel-uncertainty predictions. Thus d' additivity and standard probability summation should continue to be useful for calculation, even if we accept the channel uncertainty model as the explanation.

The uncertainty hypothesis offers parsimonious explanation of many characteristics of visual detection. This suggests that human observers do not use exact knowledge of the signal in contrast detection. This was confirmed by direct experimental test.

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[21] Nachmias and Sansbury (1974) suggested this might be true, without proving it. Nachmias and Kocher (1970) showed something analogous to d' additivity held for ROC performance of the channel-uncertainty model.

APPENDIX 1

CHARACTERIZING VISUAL NOISE

Perhaps the greatest hurdle in comparing and understanding the experiments that have studied effects of visual noise[1] is the profuse variety of terms that have been used to specify the physical characteristics of the noise.

Luminance noise is random fluctuation of luminance over time or space, or both. By "random" I mean unpredictable by even an ideal observer. This will allow comparison of actual observer performance with that of an ideal observer. In terms of this definition an image presented repetitively and predictably to an observer is not random, and hence is not noise, no matter how it was made.

While luminous stimulation always extends in two spatial dimensions as well as time, luminance noise will be described only in its dimensions of random variation. Thus we will speak of "zero-", "one-", and "two-dimensional" noise, to indicate the number of spatial dimensions over which the noise varies randomly. Noise that fluctuates randomly over time will be called "dynamic", otherwise it will be called "static". Note that this definition allows for a "static" noise pattern to be faded on and off.

A few examples may help. A grainy photograph (seen for the first time)

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[1] This usage is common now, but relatively new. Higgins and Perrin (1958) protested its arrival,

> "Strenuous objections have been raised to the extension of the term 'noise' from the field of acoustics to the fields of photography and optics, but the use of the term in communication theory to mean an unwanted signal of any description seems to be firmly established. Any extension of the term 'noise' to the photographic or visual aspects of granularity is still objectionable."

But when one treats an image as a message where does communication theory end, and optics begin?

contains static, two-dimensional noise. A grainy movie, and a "snowy" television picture, contain dynamic two-dimensional noise. Parallel bars, fluctuating in luminance randomly and independently, are an example of dynamic, one-dimensional noise. We will need to consider all of these.

The following definitions are meant to be general enough so that the terms may be used to describe noise with various numbers of dimensions of random variation. However, in order to keep the text readable many phrases and equations in what follows are cast in terms of only one dimension. The generalization to more than one dimension will sometimes be indicated when it is not obvious.

The definitions are all based on the physical description of an image as a luminance distribution over space and time. However, in part because of convention, in part because of the experimental finding that average luminance is of only secondary importance in effects of luminance noise, the following measures are defined in terms of a _contrast function_, equal to the luminance function divided by its space-time average, minus 1 (Linfoot 1964).

The variance of the contrast function will be called the contrast _power_. The square root of power is commonly used in the vision literature (e.g. Stromeyer and Julesz 1973) and is called _rms contrast_, c_{rms}, so we may represent the power by c_{rms}^2. Both of these quantities are dimensionless.

Conventional, _Michelson, contrast, c_, is half the peak-to-peak fluctuation of the contrast function. The amplitude of a sinusoidal grating may be characterized by either its Michelson contrast, or its rms contrast, which is $1/\sqrt{2}$ times its Michelson contrast. For noise, however, the Michelson contrast is much less informative than the rms contrast. Usually we will want to know the Michelson contrast of noise only to determine whether it exceeds the linear range of a display device.

Jones (1955) suggested that image noise should be characterized by its spatial-frequency spectrum. The spectrum describes how the power, c^2_{rms}, is distributed over frequency. Consider filtering an image by a narrowband filter with unit gain. The original image's (two-sided) <u>spectral</u> <u>density</u>, <u>$n^2(f)$</u> (at the center frequency, f, of the filter) is the noise power of the filtered image divided by the two-sided[2] bandwidth of the filter (in the limit as bandwidth approaches zero). "<u>spectrum</u>" is another name for the spectral density function. The integral of the spectrum over all frequencies, positive and negative, equals the noise power,

$$c^2_{rms}=\int_{-\infty}^{+\infty} n^2(f)\, df$$

$$=2 \int_{0}^{+\infty} n^2(f)\, df.$$

In general, if the spectral density is a function of k dimensions then the power is

$$c^2_{rms} = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} n^2(f_1,f_2, \dots ,f_k)\, df_1 \dots df_k$$

$$=2^k\int_{0}^{+\infty} \dots \int_{0}^{+\infty} n^2(f_1,f_2, \dots ,f_k)\, df_1 \dots df_k$$

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[2] That is, the sum of bandwidth in positive and negative frequency. For one-dimensional noise this is just twice the conventional one-sided bandwidth. For two-dimensional noise is is four times, etc.

This definition considers the spectral density to be two sided, that is, it is defined (and equal) at positive and negative frequency: $n^2(f)=n^2(-f)$. The audition literature uses the one-sided spectral density $N_0(f)$. If the noise is a function of k dimensions the two-sided power spectral density, $n^2(f)$, is related to the one-sided power spectral density, $N_0(f)$, by

$$n^2(f)=N_0(f)/2^k,\ f\geq 0.$$

To summarize, the square of the rms contrast c_{rms} is the power c_{rms}^2. $n^2(f)$ is the power spectral density of the image at each frequency. The integral of the spectral density, $n^2(f)$, over all frequencies equals the power, c_{rms}^2.

In practice most convenient sources of noise produce "white" noise. Noise is white if its spectral density is constant at all frequencies (i.e. $n^2(f) = n^2$).[3] White noise can never be realized in practice because it would require infinite power. This is not a problem because white noise is used as an input to systems of limited bandwidth; it is sufficient to make the spectral density constant from zero frequency through the highest frequency to which the observer is sensitive. Thus low-pass white noise will often be called simply "white".

Now consider measurement. Beginning in 1920 photographic scientists devised one measure after another of image noise, striving to find the best correlate of apparent graininess of photographic film. The diversification ended when Marriage and Pitts (1956) showed that all the popular measures, including the spectrum, were equivalent in the sense that any one could be calculated from any other.

The easiest and oldest objective measure of luminous fluctuation is the variance of the normalized readings by a photometer which scans the image with a sampling aperture ("normalized", as before, means divided by the overall average). For a grainy photograph a small disk is an appropriate sampling aperture. In general one must specify the sampling aperture extent in each dimension of variation of the noise. Thus, for a grainy movie one would

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[3] White Gaussian noise has the additional characteristic that the phases of all frequencies are independent.

specify the height, width and duration of the sampling aperture. The "aperture size" is the generalized volume of the aperture. For dynamic two-dimensional noise the size of an appropriate aperture would be height x width x duration; for static one-dimensional noise the aperture size would be its width.

Each luminance sample will be the average luminance in the sampling aperture, so the measured luminance function will be "smoother" than the actual luminance function. As a result the measured power may be less than the actual power, c^2_{rms}. As a result the power of the measured luminance function may be less than the power of the actual luminance function. If the aperture dimensions are small relative to one half period of every frequency contained in the noise, then the measured power will equal the actual power, c^2_{rms}.

Selwyn (1935) showed that if the fluctuations are uncorrelated from point to point then the product of the aperture area and the measured variance (of the contrast function* readings) is constant. Let us call this the aperture-power product. If there are short-range correlations, for example due to the finite size of the silver grains, then the constancy will hold only for apertures substantially larger than the range of the correlations. When the aperture-power product is specified as a constant it is implied that all large apertures used for measuring it would give the same value.

It would be historically appropriate to call the square root of the aperture-power product "granularity". Marriage and Pitts (1956) said,

> "If a piece of photographic material is uniformly exposed and processed and then examined closely it is seen to be nonuniform in transmission. The visual appearance of irregularity is called graininess; physical measurements of the fluctuation in transmission factor or in optical density are generally called measurements of granularity . . . [Selwin (1935)] proposed that granularity be measured by σ/a [where σ is the

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* To be precise, Selwin refered to density (i.e. minus logarithm of transmission) readings from a microdensitometer, not luminance readings from a photometer, but the argument is the same.

standard deviation of the normalized density readings, and a is the area of the sampling aperture]. . . . we shall call the curve of $\sigma\sqrt{a}$ as a function of a the Selwyn granularity curve, or simply the granularity, its variation with a being understood."

Defining granularity as the square-root of the aperture-power product differs from Marriage and Pitts only in being generalized to other than two dimensions, and in being based on luminance readings, rather than density or transmission readings.

White noise satisfies Selwyn's requirement that fluctuations be uncorrelated from point to point (as long as the aperture is large relative to one half period of the highest frequency in the noise). Jones (1955) showed that if the noise is white then the aperture-power product does not depend on the aperture size (within the above constraint) and is equal to the power spectral density, n^2, of the noise[5], and, conversely, if the aperture-power product does not depend on the aperture size then the noise is white. Of course, the same arguement applies to their square roots: a constant granularity is equivalent and equal to a constant n, the square root of the spectral density. The aperture-power product (or its square root, the granularity) is the easiest way to measure the spectral density of white noise.

In summary, sampling apertures much smaller than half a period of the highest noise frequency will give a constant power equal to the noise power c^2_{rms}; if the noise is white then apertures much larger than half a period of the cut-off frequency will give a constant aperture-power product equal to the spectral density, n^2.

One more description of noise should be mentioned. Several important studies used ideal image intensifiers which displayed a small brief dot (or "speck") of light for each detected photon (e.g. Rose 1948, van Meeteren and

Boogaard 1973). Since the dots are statistically independent the noise is white and may be characterized by an aperture-power product, as discussed above. The aperture-power product, and thus the spectral density, of dot noise are equal to the inverse of the dot flux.

Power or spectral density?

Power and spectral density are complementary measures. In the ideal, white noise has a constant spectral density and infinite power. It is fully characterized by its spectral density. On the other hand, the limit of the narrow-band case, a sinusoidal grating, has infinite power spectral density but

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[5] Here follows a simple proof that the aperture-power product of white noise equals its power spectral density. Suppose the spectral density, $n^2(f)$, is a constant, n^2, up to a cut-off frequency f_c and zero beyond it. The integral for total noise power is

$$c^2_{rms} = \int_{-\infty}^{+\infty} n^2(f)\, df$$

$$= \int_{-f_c}^{f_c} n^2\, df$$

$$= 2\, f_c\, n^2.$$

The noise, having cut-off frequency f_c, may be fully represented by samples spaced at intervals of $1/(2f_c)$. Furthermore those samples will be uncorrelated. The sample variance is equal to the noise power, c^2_{rms}. The average of J uncorrelated variables, each with variance c^2_{rms}, has variance c^2_{rms}/J. Thus the variance measured with a large aperture which includes J samples will be c^2_{rms}/J. The aperture-power product is the product of aperture size and measured variance:

$$[J\ 1/(2f_c)]\ [c^2_{rms}/J]$$

$$= c^2_{rms}/(2f_c)$$

$$= n^2.$$

This completes the proof. Note that this also shows that the aperture-power product is the same for all aperture sizes that are large relative to $1/(2f_c)$, a half period of the cut-off frequency, f_c.

finite power (c_{rms}^2 equals half the square of the Michelson contrast of a sinusoid). In practice we will often need to describe bandlimited white noise. If the bandwidth exceeds the bandwidth of the optics of the eye it is clear that further increases in bandwidth can have no effect on the retinal image. The noise power, c_{rms}^2, is of little consequence in this situation. Spectral density is all we would need to know. In the review of critical-band experiments it will emerge that the relevant visual bandwidth that divides broad from narrow is the one-to-two-octave bandwidth of a spatial-frequency channel, rather than the much broader bandwidth of the optics of the eye used in the argument above.

The effect of viewing distance

Since we are primarily interested in the pattern produced on the observer's retina, the lengths which appear implicitly in the above definitions (e.g. aperture size, spatial frequency, dot flux) should be measured as angles subtended at the observer's eye. Stepping away from a periodic wallpaper pattern increases its spatial frequency from the point of view of the observer, but its rms contrast and its Michelson contrast are dimensionless and independent of viewing distance. So much is obvious. Not so familiar is the dependence of contrast spectral density upon viewing distance. Consider two-dimensional static noise with spectrum $n^2(f_x,f_y)$ or or two-dimensional dynamic noise with spectrum $n^2(f_x,f_y,f_t)$. Doubling the viewing distance doubles the spatial frequencies in both horizontal and vertical dimensions. Recall that the spectral density is the power per unit (two-sided) bandwidth. The power, c_{rms}^2, is unchanged, but the spatial bandwidths are doubled. It follows that doubling the viewing distance quarters the spectral density. The new spectrum is

$$n'^2(f_x,f_y) = \frac{1}{4}\, n^2(f_x/2,f_y/2)$$

or

$$n'^2(f_x,f_y,f_t) = \frac{1}{4}\, n^2(f_x/2,f_y/2,f_t).$$

The same power has been distributed over a larger bandwidth.

This may be most easily seen by considering dot noise. We saw earlier that the spectral density of dot noise is equal to the inverse of the dot flux, and doubling the viewing distance obviously quadruples the dot flux.

For one-dimensional noise there is only one spatial bandwidth to be doubled, and so the spectral density is only halved: if the initial spectrum were $n^2(f_x)$, then doubling the viewing distance would make it $n'^2(f_x) = \frac{1}{2}\, n^2(f_x/2)$.

The effect of viewing distance may have been the first psychophysical measure of visual noise. Jones and Higgins (1945) say,

> "Some of the earliest work done on the study of the graininess of photographic deposits was reported in publications from these [Kodak Research] Laboratories (Jones and Deisch 1920, Hardy and Jones 1922). A method was used in which an enlarged image of the uniformly exposed and developed photographic material was moved away from the observer until it appeared homogeneous. . . . the relative merging distances for different samples were used to evaluate graininess. It was found possible to make measurements with a precision which gave an average deviation of ±3 percent. A similar method was used by Crabtree to compare the graininess of motion-picture films."

If the film noises were white over the bandwidth of the observer's visual optics, then the effect of increasing the viewing distance was simply to reduce the power spectral density to threshold.

NOMENCLATURE

Contrast, Power, and Spectra

Normalized luminance - the actual luminance (a function of space and time) divided by its space-time average. Dimensionless.

c - Michelson contrast or contrast may be defined as half the peak-to-peak fluctuation of the normalized luminance. Dimensionless.

c_{rms} - rms contrast, defined as the standard deviation of the normalized luminance. Dimensionless.

c^2_{rms} - contrast power (usually of noise), defined as the variance of the normalized luminance. Dimensionless.

$n^2(f)$ - (two-sided) contrast power spectral density or spectrum, usually of noise. The power per unit two-sided bandwidth. The power, c^2_{rms}, equals the integral of the spectrum, $n^2(f)$, over all frequencies (positive and negative). "Spectral density" refers to a particular value of $n^2(f)$, while spectrum refers to the entire function. Has units of one over bandwidth.

n^2 - a constant spectral density. When the spectral density $n^2(f)$ is constant over a band, and zero outside it, then the value within the band will often be represented by n^2.

$n(f)$ and n - the positive square root of the spectral density. It is proportional to the rms contrast of the noise. Has units of one over square root of bandwidth.

White noise - noise "whose spectral density is sensibly constant from zero frequency through the frequencies of interest," (Blackman and Tukey 1958).

s^2 - the signal energy, the contrast energy of the signal. It is defined as the integrated, squared deviation (of the expected signal) from the normalized mean luminance. The integral is to be taken over the entire signal extent along dimensions of noise fluctuation. In audition the preferred symbol is E, where $E=s^2$ (eg. Tanner and Birdsall 1958). It has units of size.

s - the positive square root of the signal energy. It is proportional to the contrast of the signal. Has units of square root of size.

Aperture - a sampling window for measuring noise or signal characteristics. As such it must have extent only in the dimensions of noise variation (eg. it must have a duration if the noise is dynamic, it must not if the noise is static).

Aperture size - the generalized volume of the aperture (eg. height x width x duration, for an aperture extending in two dimensions and time)

Aperture-power product - product of the aperture size and the power (ie. variance of the normalized luminance) measured through it. If the noise is white the aperture-power product is constant for all apertures which are large relative to half a period of the highest frequency present. Has units of size.

Granularity - the positive square root of the aperture-power product. Originally defined by Selwyn (1935) in a slightly different way.

Dot rate - the average number of statistically-independent, identical spots of light per unit aperture. Its inverse is the spectral density of the dot noise. Has units of one over size.

Critical Spectral Density - The spectral density of a noise mask which raises the squared threshold to twice its zero-noise value. All relevant studies have reported that the square of the contrast threshold is proportional to the sum of the actual and critical spectral densities of noise masks. Has the same units as spectral density.

Transduction Efficiency - the ratio of the photon noise, $n^2_{photons}$ to the observer's equivalent noise, measured in practice as $n^2_{critical}$:

$$TE = (n^2_{photons}/n^2_{critical}).$$

Signal detection theory

s/n - signal-to-noise ratio. The terms s and and n were defined above. When a signal is added to white Gausian noise whose bandwidth includes the entire signal bandwidth then the detectability of the signal by an ideal observer which knows the signal exactly is determined only by this ratio. Most of the communication theory and audition literature represents this as $\sqrt{(2E/N_0)}$, where E is the signal energy (ie. s^2), and N_0 is the one-sided spectral density of the noise (ie. $n^2=N_0/2$, assuming the noise is a function of time only, as is usually the case in audition). If the noise were a function of k dimensions the two-sided spectral density would be $N_0/2^k$, and the signal-to-noise ratio would be $\sqrt{(2^k E/N_0)}$. Barlow (1978) represented the signal-to-noise ratio by d'_I (ie. $d'_I \equiv s/n$). Dimensionless.

d' - the signal-to-noise ratio (defined above) that an ideal observer would require to match a given performance by an observer under study (Tanner and Birdsall 1958). For 2-alternative forced-choice responses, if the signal is known exactly, d' equals $\sqrt{2}$ times the normal deviate corresponding to the proportion correct, P(C) (Green and Swets 1966). Dimensionless.

Efficiency - the ratio $(d')^2/(s/n)^2$ (Tanner and Birdsall 1958). Also called "detective efficiency". Dimensionless.

Quantum Efficiency - the special case of efficiency when there is no luminance noise. Often defined as the fraction of the corneal quanta which an ideal would have required to perform as well as the observer under study (Barlow 1958b, Jones 1959, Barlow 1962a,b). Dimensionless.

APPENDIX 2

CROSS-CORRELATION AND ENERGY DETECTORS

Evidence for independent detection of spatial frequencies differing by more than an octave makes it reasonable to suppose that the eye, like the ear, may be considered a bank of parallel detectors, each sensitive to only a band of frequencies. But what sort of detector?.

Two models will be considered. Firstly, coherent detection: a cross-correlation detector integrates the product of the input and the expected signal. This is ideal when the signal is known exactly. Secondly, incoherent detection: an energy detector integrates the square of the filtered input over the known signal extent and duration. Without the input filter the energy detector provides ideal detection of a noise increment, thus it represents an extreme in its high signal uncertainty (Green and Swets 1966).

The operation of cross-correlation is equivalent to convolution by the time-reversed expected-signal, and evaluating the result at the origin. Thus both detectors may be characterized as having an input filter with a modulation transfer function $|H(f)|$. Only the relative gain matters so arbitrarily take $|H(f)|$ to be unity at its peak.

The signal-to-noise ratio at the outputs of the cross-correlation and energy detectors will now be derived.

Cross-correlation detection

The cross-correlation detector is linear so the mean output will depend only on the signal, and the variance of the output will depend only on the noise. We will assume that the noise is Fourier-series band limited so that we may decompose the noise into a Fourier series whose components have stochastically independent amplitudes. The number of these components is twice the product of the noise bandwidth and its duration (i.e. spatial extent). Note that the relevant extent is the lesser of the physical extent and the extent of the channel's receptive field. The observed bandwidth of the channels requires a receptive field whose extent is at least two periods of the center frequency. Thus 1 c/deg bandwidth noise masking a 4 c/deg channel may be decomposed into a Fourier series of at least

$$2 \times 1 \text{ c/deg} \times 2/(4 \text{ c/deg}) = 1 \text{ component.}$$

The contribution of each noise frequency to the variance at the detector output is proportional to the product of the noise power spectral density $n_0(f)^2$, and the square of the filter gain at that frequency, $|H(f)|^2$. The total variance will be the sum of the variances over all the frequencies.

$$\sigma^2 \propto \int n_0(f)^2 \, |H(f)|^2 \, df$$

The mean difference due to the signal's presence is proportionl to the signal amplitude, c. The ratio of the mean difference to the standard deviation is

$$\begin{aligned} d' &= (\mu_{SN} - \mu_N)/\sigma \\ &\propto c/\left(\int n_0^2 \, |H(f)|^2 \, df \right)^{1/2}. \end{aligned}$$

The constant of proportionality may be obtained by considering noise which (in the limit) has the same spectrum as the signal, but random phase. If the noise and signal spectra are equal, the phase specificity of the cross-correlation detector will give a d' of $\surd 2$. Thus

$$d' = \sqrt{2}\, c/\left(\int n_0^2\, |H(f)|^2\, df \right)^{1/2}.$$

Energy detection

In audition there is some evidence that detection is effectively mediated by energy detectors at the output of each filter. Until recently it was erroneously assumed that the performance of an energy detector would depend only on the ratios of signal to noise energies which it received. However Green and Swets (1966) and Patterson and Henning (1977) have pointed out that the performance of an energy detector is determined by the ratio of signal energy to the standard deviation of the noise energy.

The energy detector is nonlinear and therefore more difficult to analyze than the cross-correlation detector. The mean difference in energy output is approximately[6] the energy of the signal $(1/2)c^2T$ (where T is the known signal duration). The variance of the output energy is a sum of contributions from all the Fourier components of the noise; the variance of each component is proportional to the product of $n_0^4(f)$ and $|H(f)|^4$. So

$$d' \propto c^2T/\left(\int n_0^4\, H(f)^4\, df \right)^{1/2}$$

[6] Patterson and Henning (1977) found that the nonlinear interaction of the signal and noise results in a noncentral Chi-square distribution at the detector output. However if the ratio of the signal power times duration to the noise power spectral density is greater than 3 (and the auditory thresholds they report all have ratios in excess of 10) then ignoring the signal-dependent component of the variance leads to underestimating the filter gain $|H(f)|$ by less than 5%.

Solving the derived equations for the contrast at threshold shows that they have a very similar form.

$$c = \sqrt{2}\, d' \left(\int n_0^2 \, |H(f)|^2 \, df \right)^{1/2} \quad \text{(cross-correlation detector)}$$

$$c = k \sqrt{(d'T)} \left(\int n_0^4 \, |H(f)|^4 \, df \right)^{1/4} \quad \text{(energy detector)}$$

G: what is meant by threshold criterion? Isn't T just he signal duration, and thus known to be constant? It seems reasonable to suppose that d' and T are constant for a fixed threshold criterion (i.e. 82% correct), so that relative threshold elevation will not depend on these unknown parameters. Let $c_{all\text{-}pass}$ represent the threshold contrast in all-pass white noise whose noise spectral density is $n^2_{all\text{-}pass}$. Normalizing by the all-pass threshold gives

$$\frac{c}{c_{all\text{-}pass}} = \frac{\{ \int n_0^{2B} \, |H(f)|^{2B} \, df\}^{1/(2B)}}{\{ \int n_{all\text{-}pass}^{2B} \, |H(f)|^{2B} \, df\}^{1/(2B)}}$$

where 1/(2B) is the index of summation: 1/2 for the cross-correlation detector, and 1/4 for the energy detector.

We may try to understand this last equation. If the output of a detector increases as the B-th power of its input then its output variance will increase, approximately, as the 2B-th power of the amplitude of any input noise. Neglecting distortion products, the frequency components of the noise are independent so the total output variance will be a sum of the variances due to each of the components of the noise. The output amplitude of the signal is the B-th pwer of the input amplitude, so the signal amplitude required for a given d' will be proportional to the 1/B power of the output standard deviation, or the 1/(2B) power of the output variance.

The two detectors differ in several respects. We have seen that threshold is proportional to an integral over noise frequencies with exponent 2B. In the cross-correlation detector d' is proportional to the signal amplitude c. In the energy detector d' is proportional to the signal power c^2. In both cases d' is proportional to c^B. What is the experimental finding? For 2IFC d' is equal to $\sqrt{2}$ times the normal deviate corresponding to the proportion correct. For detection of gratings, Nachmias and Sansbury (1974) found that d' increased as $c^{2.2}$ for one observer, and $c^{2.9}$ for another, and Foley and Legge (1981) report exponents between 2.11 and 3.04 for two observers at three different spatial frequencies (.5, 2, and 8 c/deg). Neither the cross-correlation nor the energy detector has a psychometric function as steep as these. The above derivations are general enough to allow for B to be say, 3. Alternatively, channel uncertainty among a large number of cross-correlation detectors (or energy detectors for that matter) would generate psychometric functions of the required steepness, but that is the topic of Chapter 5.

APPENDIX 3

THE EFFECTS OF HETERODYNE NOISE

Heterodyne means that the two sidebands are symmetric about the carrier frequency, i.e. the components at frequencies $\omega_c-\omega$ and $\omega_c+\omega$ have even-symmetric amplitude and odd-symmetric phases[7]. The most obvious manifestation of this symmetry is that the heterodyne noise has zero crossings at all the zero crossings of the original carrier (Spiegel 1979).

Correlation between the sidebands complicates the analysis of the total variance produced in a channel because it is normally assumed that each frequency component of the noise makes an independent contribution to the variance in the channel. Nevertheless this has been the technique most often used in audition to produce narrowband noise (e.g. Greenwood 1961).

In audition the channels are thought to be non-phase-specific energy detectors, so the phase symmetry is not a concern. However the amplitude symmetry would cause a channel symmetric about the carrier frequency to receive equal contributions from each sideband, so the standard deviations would add, instead of the variances, as would happen if the components were independent.

In vision the spatial-frequency channels may be phase specific. I will now calculate the response of a cross-correlation detector to a symmetric pair of components of heterodyne noise. The input is

$$\alpha/2\ \{\cos(\omega_c-\omega t-\phi) + \cos(\omega_c+\omega t+\phi)\}$$

The output will depend only on the amplitude and phase of the expected signal

[7] Phases are taken relative to an arbitrary origin at which the cosine carrier is one.

at those frequencies. Let us represent the relevant components of the expected signal as

$$\beta \cos(\omega_c-\omega t+\theta) + \beta'\cos(\omega_c+\omega t+\theta')$$

where β and β' are taken as positive. We want to know how the amplitudes β and β' and the phases θ and θ' affect the output. The cross-correlation will be

$$\alpha/2\{\beta \cos(\theta+\phi) + \beta'\cos(\theta'-\phi)\}$$
$$=\alpha/2\{[\beta \cos(\theta)+\beta'\cos(\theta')]\cos(\phi) - [\beta \sin(\theta)-\beta'\sin(\theta')]\sin(\phi)\}$$

Remembering that α and ϕ are random variables, we can see that the output will be identically zero only if

$$[\beta \cos(\theta)+\beta'\cos(\theta')]=0$$

and

$$[\beta \sin(\theta)-\beta'\sin(\theta')]=0.$$

Alternatively the total will be twice that due to either sideband when

$$\beta \cos(\theta)=\beta'\cos(\theta')$$

and

$$\beta \sin(\theta)=-\beta'\sin(\theta').$$

Both alternatives require that

$$\beta=\beta'.$$

Thus these extremes can occur only when the spectrum of the expected signal is symmetric about the center of the heterodyne noise. The output will be zero if

$$\theta+\theta'=\pi,$$

and the double-sideband (i.e. total) output will be double the single-sideband output if

$$\theta=-\theta'.$$

That is, the response of a cross-correlation detector to heterodyne noise will differ most from its response to truely-white narrow-band noise of the same spectrum if

1. the spectrum of the expected signal is symmetric about the carrier frequency of the heterodyne noise, and either

2a. the phases of the expected signal are odd-symmetric about the carrier frequency, in which case the output will be twice that due to just one sideband, or

2b. the phases at frequencies symmetric about the carrier are complementary angles, in which case the output will be identically zero.

On the positive side it should be noted that if the spectrum of the expected signal is very asymmetric with respect to the carrier frequency, so that the two sidebands of the noise receive very different attenuations, then the contribution from one sideband or the other will dominate and the symmetry of the heterodyne noise will not matter. In practice this will mean that the symmetry of heterodyne noise may safely be ignored, except when the noise is centered on the channel peak, where channel sensitivity may be overestimated by as much as $\sqrt{2}$, or, for phase-specific detection, may be grossly underestimated, depending on the phase relations of the expected signal and the carrier of the heterodyne noise.

Conclusion

The symmetry of heterodyne noise may lead to overestimating the peak sensitivity of an energy detector by as much as 2^2, but will not cause an underestimate. For a cross-correlation detector the peak sensitivity may be overestimated by as much as $\surd 2$, or may be grossly underestimated.

APPENDIX 4

GENERATING NARROW-BAND NOISE

The obvious way to produce narrow-band noise would be to pass white noise through a bandpass filter. However, for ease of analysis it is desireable to have very steep fall off at the edges of the band. It is difficult to build an adjustable bandpass filter with skirts as steep as desired. For conventional filter designs the steepness of the fall off is inversely proportional to the cut-off frequency. Three techniques will described which achieve a steep fall off, independent of cut-off frequency, by doing the filtering at a low frequency, and then shifting the noise band up to the desired center frequency. The first technique is simpler but produces heterodyne noise whose frequency components are not completely independent. The third technique is just an unconventional bandpass filter. All these techniques are known, but never seem to have been collected in one place before.

Analysis of the techniques is facilitated by considering their action on an arbitrary frequency component of the noise. Over any interval of finite duration T we may represent white noise by a Fourier series whose components have stochastically independent amplitudes. Let ω be a multiple of 1/T. At this frequency we have

$$u \cos(\omega t) + u' \sin(\omega t)$$

where u and u' are stochastically independent, zero-mean, and Gaussian, and have equal variance. We may change coordinates to

$$\alpha = \sqrt{(u^2 + u'^2)}$$

$$\phi = -\arctan(u'/u),$$

so

$$u \cos(\omega t) + u' \sin(\omega t) = \alpha \cos(\omega t + \phi).$$

The new random variables α and ϕ are stochastically independent, ϕ has a uniform distribution, and α has a Rayleigh distribution.

AM

Greenwood (1961) described the simplest technique: (suppressed-carrier) amplitude modulation (AM), which produces heterodyne noise. A low-pass filter is used to select a narrow band of noise centered at zero frequency, then this is multiplied by a carrier, e.g. $\cos(\omega_c t)$, of the desired center frequency, ω_c. Consider the effect of this operation on an arbitrary frequency component of the noise with frequency ω, amplitude α, and phase ϕ:

$$\alpha \cos(\omega t + \phi).$$

The output will be[8]

$$\cos(\omega_c t)\ \alpha \cos(\omega t+\phi)=\alpha/2\ \{(\cos(\omega_c t-\omega t-\phi) + \cos(\omega_c t+\omega t+\phi)\}$$

$$\text{if } |\omega|<\omega_0, \text{ and zero otherwise.}$$

The single input frequency produced two output frequencies with equal amplitudes and inverted phases (i.e. $-\phi$ and ϕ). Because each frequency component of the input produces two symmetric frequency components in the output the resulting heterodyne noise has twice the bandwidth of the low-pass

[8] Most of the algebra in this appendix is just repeated application of three trigonometric identities:

$$\cos(a)\cos(b)=1/2\ \{\cos(a+b) + \cos(a-b)\}$$
$$\sin(a)\cos(b)=1/2\ \{\sin(a+b) + \sin(a-b)\}$$
$$\sin(a)\sin(b)=1/2\ \{-\cos(a+b) + \cos(a-b)\}$$

filter.

Unfortunately the effects of heterodyne noise can be considerably more difficult to analyze than truely white noise (see "The effects of heterodyne noise"), more than compensating for the relative ease with which it may be generated. The following techniques produce truely-white narrow-band noise.

Quadrature AM

Spiegel (1979) described a technique which may be called quadrature AM produces a narrow band of noise with no symmetry. Take two independent noise sources. Pass both through identical low-pass filters. Multiply one by a sine carrier, multiply the other by a cosine carrier, and then add them[9]. We start with

$$\alpha \cos(\omega t+\phi)$$

and

$$\alpha'\cos(\omega t+\phi')$$

where α, α', ϕ, and ϕ' are all stochastically independent. The phases have a uniform distribution, and the amplitudes have a Rayleigh distribution. Multiplying by the carriers and adding gives us

$$\alpha \cos(\omega t+\phi)\cos(\omega_c t) + \alpha'\cos(\omega t+\phi')\sin(\omega_c t)$$

$$=\alpha/2\{\cos(\omega_c t-\omega t-\phi)+\cos(\omega_c t+\omega t+\phi)\} + \alpha'/2\{\sin(\omega_c t-\omega t-\phi')+\sin(\omega_c t+\omega t+\phi')\}.$$

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[9] On a raster-type display this may be achieved by using one noise source and using the sine and cosine carriers on alternate frames of the display, relying on the observer's eye to sum them.

Substituting $\phi''=\phi'+\pi/2$ gives

$$=1/2\{\alpha\cos(\omega_c t-\omega t-\phi)+\alpha'\cos(\omega_c t-\omega t-\phi'')\} + 1/2\{\alpha\cos(\omega_c t+\omega t+\phi)-\alpha'\cos(\omega_c t+\omega t+\phi'')\}.$$

Define four new random variables,

$$x=\alpha\cos(\phi)$$

$$y=\alpha\sin(\phi)$$

$$x'=\alpha'\cos(\phi'')$$

$$y'=\alpha'\sin(\phi'').$$

These are all stochastically independent, zero mean, and Gaussian. In terms of these new variables we have

$$=1/2\{(x+x')\cos(\omega_c t-\omega t)+(y+y')\sin(\omega_c t-\omega t)+(x-x')\cos(\omega_c t+\omega t)+(-y+y')\sin(\omega_c t-\omega t)\}.$$

The amplitudes, $(x+x')$, $(x-x')$, $(y+y')$, and $(-y+y')$, are stochastically independent, zero mean, and Gaussian. That completes the analysis.

This method requires two independent noise sources, two filters, two carriers in quadrature phase, and two multipliers. Fortunately some situations allow multiplexing one noise source, filter, and multiplier between the two carriers. When that is not possible the following technique may be preferable as it requires only one noise source, at the expense of using more multipliers.

General purpose band-pass filter

It is possible to use heterodyne techniques to make a steep fall-off bandpass filter. I do not know who first devised the following technique; for a discussion of methods of single-sideband modulation see Panter (1965). In order to avoid introducing symmetries the circuit has two pathways, one uses a sine carrier, the other uses a cosine carrier. In each pathway the input spectrum is shifted down, low-pass filtered, and shifted back up. The output is the sum of the outputs of the two pathways. All shifts are performed by multiplying by the carrier frequency which also is the center frequency of the resulting passband. Again the resulting passband has twice the bandwidth of the low-pass filter. The circuit is linear so it is sufficient to determine its output for an input of arbitrary frequency and phase:

(1) $\cos(\omega t+p)$, where $\omega>0$.

First we multiply by the cosine and sine carriers:

(2) $\cos(\omega t+\phi)\cos(\omega_c t)=1/2\,\{\cos(\omega_c t+\omega t+\phi)+\cos(\omega_c t-\omega t-\phi)\}$

(2') $\cos(\omega t+\phi)\sin(\omega_c t)=1/2\,\{\sin(\omega_c t+\omega t+\phi)+\sin(\omega_c t-\omega t-\phi)\}$

Let the cut-off frequency of the low pass filter be ω_0, where $0<\omega_0<\omega_c$. Then after filtering we have

(3) $1/2\cos(\omega_c t-\omega t-\phi)$

and

(3') $1/2\sin(\omega_c t-\omega t-\phi)$ for $|\omega_c-\omega|<\omega_0$, and zero otherwise.

Multiplying by the cosine and sine carriers again gives

(4) $1/4\,\{\cos(2\omega_c t-\omega t-\phi)+\cos(\omega t+\phi)\}$

and

(4') $1/4\,\{-\cos(2\omega_c t-\omega t-\phi)+\cos(\omega t+\phi)\}$ for $|\omega_c-\omega|<\omega_0$, and zero otherwise.

Finally, the sum of (4) and (4') is

(5) $1/2 \cos(\omega t+\phi)$ for $|\omega_c-\omega|<\omega_0$, and zero otherwise.

Comparing the output, (5), with the input, (1), shows that the circuit just halves the amplitude of frequencies inside the passband and stops frequencies outside the passband. No symmetries are introduced, and the steepness of cut-off is independent of the center frequency of the pass band.

Summary

Heterodyne noise is easy to produce, but at least in theory, the symmetry of its sidebands can affect its effectiveness as a mask. Two techniques have been described which produce truely-white narrow-band noise. Both techniques require two carriers in quadrature phase. The best choice of method will depend on the purpose of the experiment and the nature of the dimension along which the noise varies.

APPENDIX 5

OFF-FREQUENCY LOOKING

FITTING THE CURVES

The fits used a "flat-top power function", but the following argument will be easier to understand if we first consider a "flat-top exponential". Consider a channel with a flat top of width f_h-f_l, and negative exponential skirts, $e^{-a|f-f_l|}$ and $e^{-b|f-f_h|}$. Let us call this a flat-top exponential. Note that the skirts would be straight on a graph of log gain versus linear frequency. There are four free parameters; a and b determine the steepness of the skirts and f_l and f_h determine the edges of the flat top. This shape may only crudely approximate the actual channel sensitivity, but most of the measureable properties of a bandpass filter depend only on the width of its passband and the steepness of its skirts. Since the width of the flat top, f_h-f_l, is free to vary, this model includes a peaked exponential as a special case.

By assumption, for each noise mask, threshold is determined by the channel with the best s/n. To find this channel Patterson and Nimmo-Smith used an iterative technique. For each set of channel shape parameters they found the position of maximum s/n, and compared the predicted threshold with their data. By iteration they found the most likely channel shape. My model allows a shortcut.

Suppose a flat-top exponential filter is placed so that both a sinusoidal signal and a band of noise are passed by one skirt, $e^{-a|f-f_l|}$. Suppose the signal frequency f_s is greater than f_l, and consider shifting the filter towards the signal a small amount Δf. Before the shift the signal gain was $e^{-a(f_s-f_l)}$. After the shift the signal gain would be $e^{-a(f_s-f_l-\Delta f)}$. The net amplification caused by the shift would be $e^{a\,\Delta f}$. Similarly the filter gain over the whole noise band would be amplified by $e^{a\,\Delta f}$. The signal-to-noise ratio would be unchanged. Pushing the filter towards the signal increases the gain for the signal and noise in direct proportion, until the flat top reachs the signal or noise.

Suppose the top reaches the signal first. Pushing further would amplify the noise, but not the signal. The s/n at this position is equal or better than the s/n at all other positions.

Suppose the top reaches the noise first. Pushing further amplifies the signal and noise but the noise is amplified in less than direct porportion, until the top reaches the signal. What position maximizes the signal-to-noise ratio? If the signal is outside the noise band, the top of the filter should be placed on the opposite side of the signal as the noise. If the noise is low pass, or high pass, so that only one edge of the noise band need be considered, the top of the filter should be placed on the opposite side of the signal as the center of the noise band. Finally, if the signal is inside a finite band of noise then the optimal placement depends on the relative steepnesses of the two filter skirts.

Most of my data were collected with low-pass or high-pass noise, or narrow bands of noise which did not include the signal frequency. Assuming the flat-top exponential shape, the thresholds were determined by only two channels: one with its top just below the signal frequency when the center of the noise was above, and one with its top just above the signal frequency when the center of the noise was below. By this interpretation the low pass masking thresholds are all from the high channel, and the high-pass masking results are all from the low channel. On the other hand the narrowband masking results directly outline the upper skirt of the lower channel, and the lower skirt of the upper channel. The narrowband masking results contain no clue as to the width of the top of the channel.

The analysis so far has used a linear frequency axis, because the noise had equal power in equal width bands of linear frequency, however existing evidence that channel width is about an octave or two independent of frequency suggests that our model shape should be described on a log frequency axis. Therefore I fit the data with a flat top and power-law skirts, which are straight when plotted as log gain versus log frequency. A power law is steepest near the top so the above conclusions still apply: the low-pass results should all be from one channel, and the high pass results should all be from another.

APPENDIX 6

THE EFFECT OF UNCERTAINTY:

DETECTING A SIGNAL AT ONE OF TEN-THOUSAND POSSIBLE TIMES AND PLACES[10]

Consider detection of a known signal at a specified time and place. That is how most of us measure a threshold: the same signal is presented in trial after trial. Every opportunity is usually taken to teach the observer exactly what he is being asked to detect, including where it will appear, and when, both as a fixed delay after requesting a trial, and by a simultaneous audible tone. At the end of each trial the observer thinks about what he saw and responds. This effort to provide exact knowledge of the signal raises a question. Does the observer use knowledge of the signal to ignore irrelevant sensations which could not have been due to the signal, in order to give more accurate responses and thus lower his threshold? If the observer knows the signal can only occur at a certain time and place can he ignore other times and places? How helpful is it to know the signal exactly? Existing theories of visual detection give three incompatible answers.

First, probability summation models usually assume individual units called "detectors" that are never active in the absence of a signal. This assumption is usually called a "high threshold". The high-threshold assumption implies that the observer does not need the exact knowledge that is usually provided because the detector units never give false alarms. In other words there are no irrelevant sensations to ignore.

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[10] This paper was presented at the 1981 meeting of the Association for Research in Vision and Ophthalmology, in Orlando, Florida (Pelli, 1981).

At the other extreme the Theory of Signal Detectability has shown that the ideal way to detect a known signal is to cross-correlate by the signal itself (Peterson, Birdsall, and Fox 1954). Various discrepancies between such a cross-correlation model and actual human performance have led to variants such as incorporating a nonlinear transformation, and supposing that we only have a fixed set of weighting functions available and that we use the one most similar to the signal. All these models require prior knowledge of the signal in order to choose the best weighting function. Without prior knowledge of the signal, simple cross-correlation would be inefficient and performance would suffer greatly.

Halfway in between these extremes lies the hypothethis of high intrinsic uncertainty. This too was inspired by developments in the Theory of Signal Detectability. In many ways human detection of a known signal is similar to ideal detection of one of many possible signals. According to the uncertainty hypothesis, the observer has a very limited ability to use prior knowledge, that is, he is intrinsically uncertain.

Let me recap. According to the high-threshold hypothesis, exact knowledge of the signal is superfluous. For the cross-correlation hypothesis exact knowledge of the signal is crucial. According to the uncertainty hypothesis attempts to provide knowledge of the signal provide only a small benefit to the observer. To test these predictions I measured the effect of depriving the observer of exact knowledge of the signal.

The signal was a brief, thin, bright, vertical bar on a large uniform background with a luminance of 340 cd/m^2. It was $\frac{1}{4}^{\circ}$ wide and lasted for 20 msecs. (The display was 27° wide and about half as high.) In the center of the display was a large fixation cross, which the observer was always instructed to fixate. In all cases the data are the responses from two-interval forced choice trials with feedback. The thresholds I will report are maximum likelihood estimates of the contrast required for 82% correct.

The first phase of the experiment measured contrast thresholds at 2^o intervals across the visual field. In order to provide the observer with exact knowledge of the signal, moveable large arrows above and below the display indicated exactly where the signal would appear, and each interval of the 2IFC trial was marked by a 20 msec beep which was simultaneous with the 20 msec signal.

Figure 1: threshold profiles for DP and DK

This slide shows the contrast threshold at each eccentricity for two observers. Viewing was binocular. The blind spots would be off the graph. Each point is an independent threshold estimate based on 50 trials. The kinky lines through the data connect up the means at each eccentricity for each observer. Those lines were taken as the observer's threshold profile over this part of the visual field. In the rest of the experiments all contrasts will be relative to these nominal threshold profiles for each observer.

In the main phase of the experiment, the signal could appear, randomly, at any of the 97 contiguous locations from -12 to +12 degrees eccentricity, and each interval of the 2IFC procedure was accompanied by a 2000 msec beep and the 20 msec signal could occur at any time during the beep. Since the signal could appear at any of 97 places and 100 times, there were 9,700 possible times and places. The staircase procedure controlled the contrast of the signal relative to the nominal threshold profile.

Figure 1

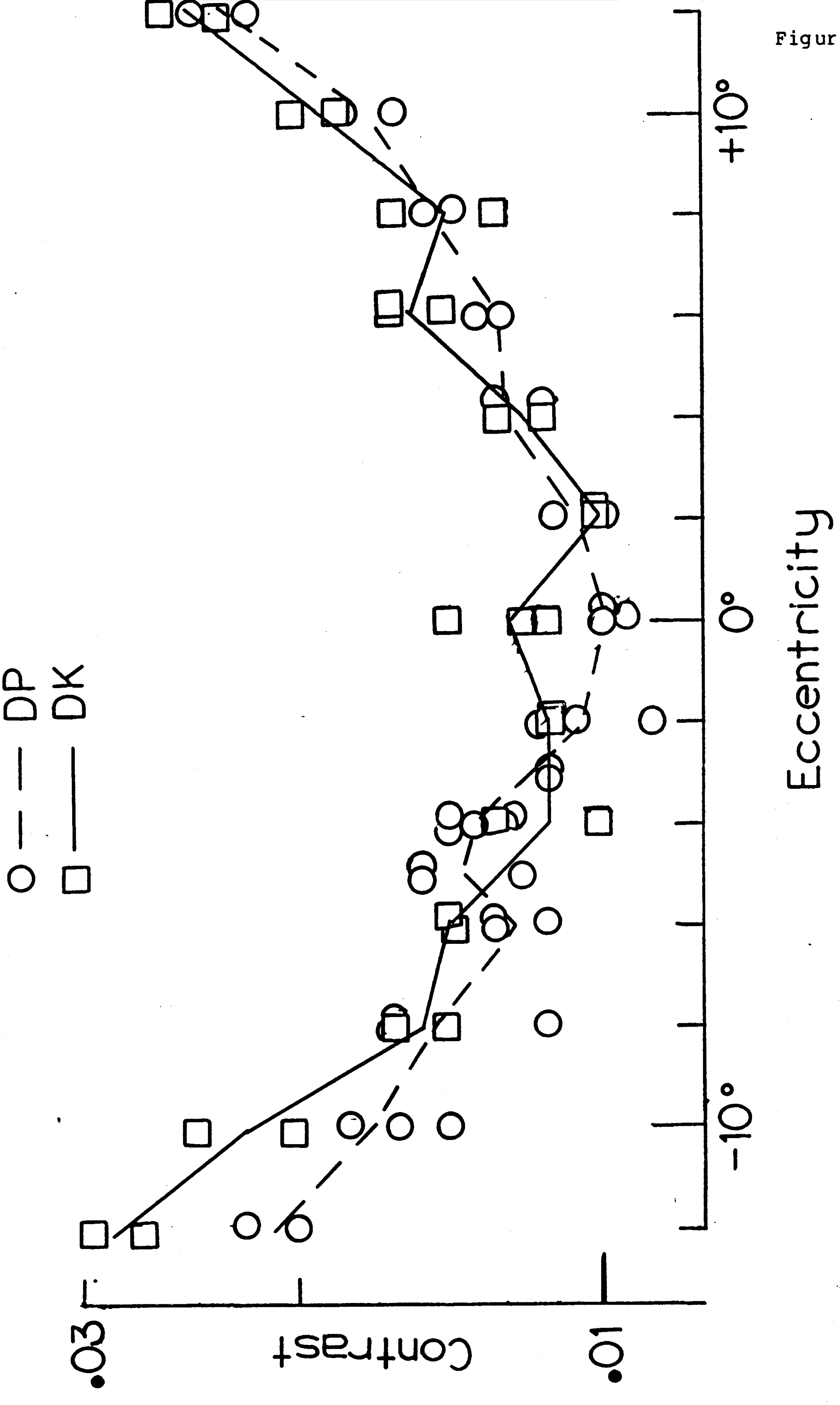

Figure 2: DP, M=9,700

M is the degree of uncertainty introduced experimentally, 9,700 in this case. The ordinate on the right is proportion correct. The abscissa is contrast relative to the nominal threshold profile. The left ordinate is d' and is labeled $\sqrt{2}z$.[11] First consider the X's, on the right. They show the proportion correct obtained for detection at each contrast.

There are different numbers of trials at each contrast. The vertical bars are 95% confidence intervals. The solid curve is the maximum likelihood, two-parameter, fit by a Quick (1974) function. Detection threshold, that is, the contrast for 82% correct is 1.4.

When I did this experiment it occurred to me that in addition to detection it would be interesting to also measure contrast discrimination. Lasely and Cohn (1981) have pointed out that uncertainty might be expected to have much less effect on discrimination than on detection. In a contrast discrimination experiment the stimulus appears in both intervals, but at different contrasts. The lower contrast is the pedestal contrast and is the same in each trial. The

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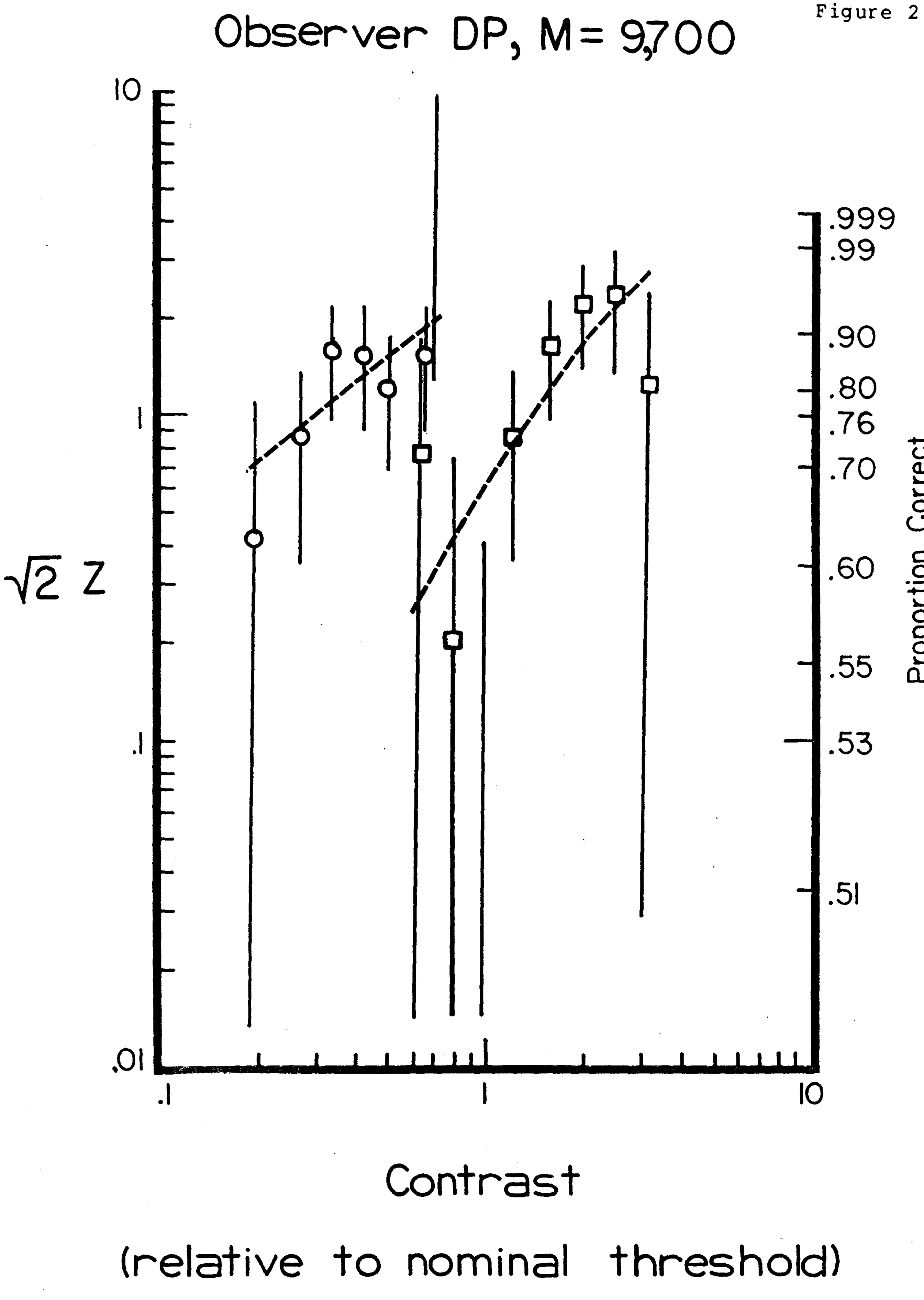
[11] This is d' as defined by Green and Swets (1966). As defined by Tanner and Birdsall (1958), d' is the signal-to-noise ratio that would be required by an ideal to perform as well as the observer under study did. Since the signal was experimentally uncertain the ideal would be a 9,700-channel-uncertainty model, whose performance, as measured by $\sqrt{2}z$, rises as the 3.5 power of contrast:

$$\sqrt{2}z \propto c^{3.5},$$

as shown in Figure 5. Figure 6 shows that in the uncertainty condition both human and ideal performance rise as the same 3.5 power of contrast, implying that, as defined by Tanner and Birdsall (1958), the obsever's d' is proportional to contrast:

$$d' \propto c.$$


Observer DP, M= 9,700
10
1
.1
.01
$\sqrt{2}$ Z
.999
.99
.90
.80
.76
.70
.60
.55
.53
.51
Proportion Correct
.1
1
10
Contrast
(relative to nominal threshold)

higher contrast is higher by an amount called the difference contrast.

The O's show discrimination of difference as a function of difference contrast, relative to the nominal threshold profile. In order to make the conditions of contrast detection (the X's) and discrimination (the O's) as comparable as possible the two conditions were randomly interleaved. The observers did not know whether a given trial was contrast discrimination or detection; he was instructed to always choose the interval with higher contrast.

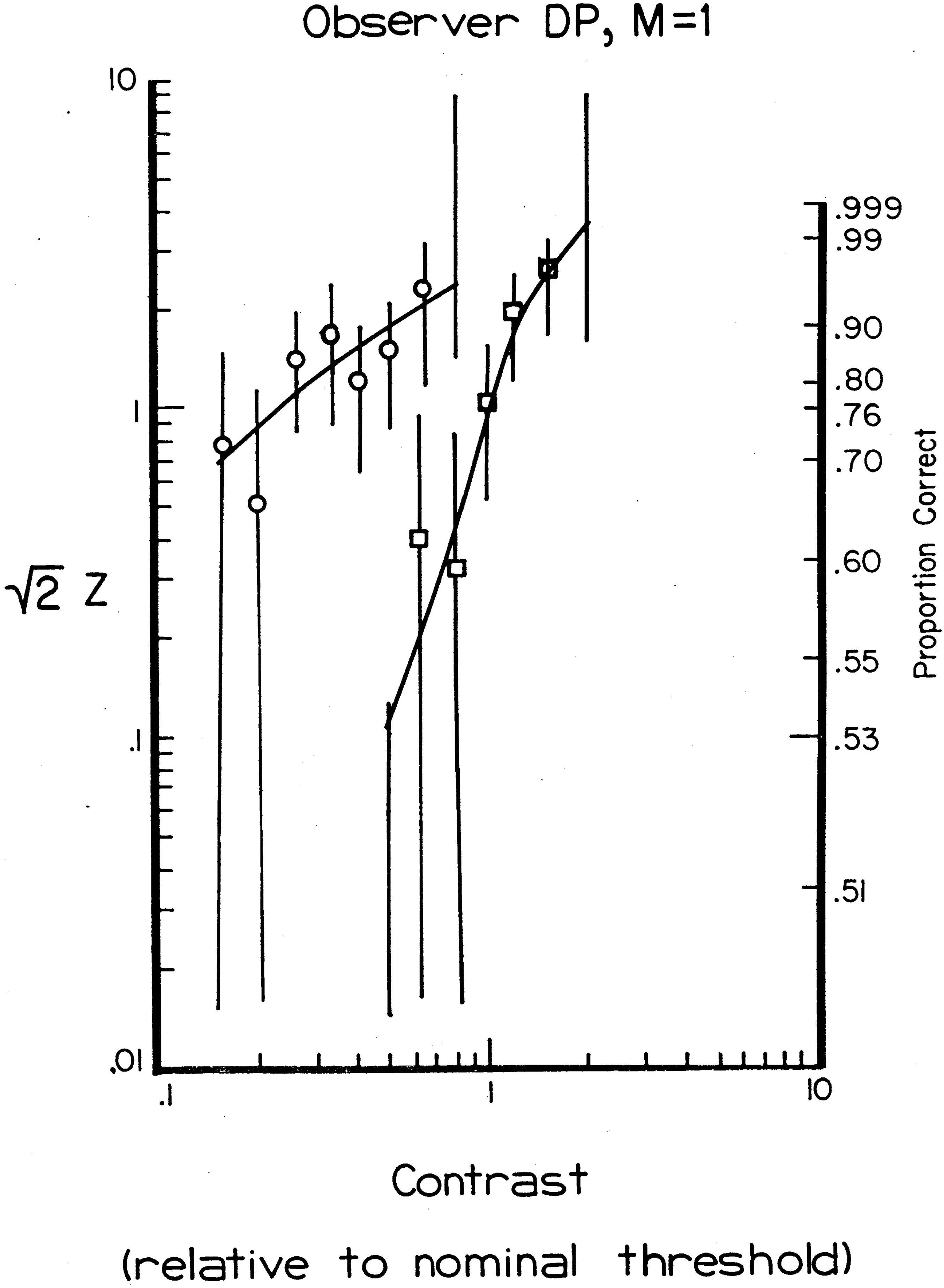
In contrast discrimination trials the stimulus was presented at the same randomly determined time and place in both intervals.

These thresholds in uncertainty are based on contrasts relative to the nominal threshold profile at many eccentricities. Samll inaccuracies in the nominal threshold profile might be systematically biasing these thresholds. To control for this possibility I measured a similar threshold for the no-uncertainty condition.

Figure 3: DP, M=1

The same procedure was used except that the beep for each interval was reduced from 2000 msec to 20 msec and coincided with the signal, and the signal, instead of appearing randomly at any place on the display, appeared, consecutively, at each position on the display. In the first 5 or 6 trials it appeared at -12° (near the left edge of the display), in the next 5 or 6 trials it appeared at -11.75° and so on until in the last 5 or 6 trials it appeared at $+12^{\circ}$ (near the right edge of the display). Pointers above and below the display indicated where the signal would occur, and were adjusted every few

Figure 3



Observer DP, M=1
10
1
.1
.01
$\sqrt{2}$ Z
.999
.99
.90
.80
.76
.70
.60
.55
.53
.51
Proportion Correct
.1
1
10
Contrast
(relative to nominal threshold)

trials to keep up with the signal. As before the observer fixated the fixation cross continuously, and all contrasts were relative to the nominal threshold profile.

These results have the familiar form described by Nachmias and Kocher (1970): discrimination is better than detection. Detection has a threshold of .9 and a steep psychometric funcion. Discrimination has a lower threshold, .4, and its psychometric function has approximately unity slope in these coordinates.

Before comparing the results with and without uncertainty I would like to ask for your indulgence to briefly consider what the Theory of Signal Detectability has to say about effects of uncertainty on ideal performance. Ideal performance for detection of a known signal is shown on the next slide.

Figure 4: ideal, M=1

As before the ordinate is d'.[12] The abscissa is the signal-to-noise ratio, which we may assume is proportional to contrast. In these coordinates the psychometric function has unity slope. The psychometric function is the same for detection and discrimination.

Figure 5: ideal, M=10,000

The graph on the right shows the ideal performance for detection of one of 10,000 orthogonal signals.[13] Relative to the no-uncertainty case we just saw, threshold is higher and the function is steeper. On the left is the ideal

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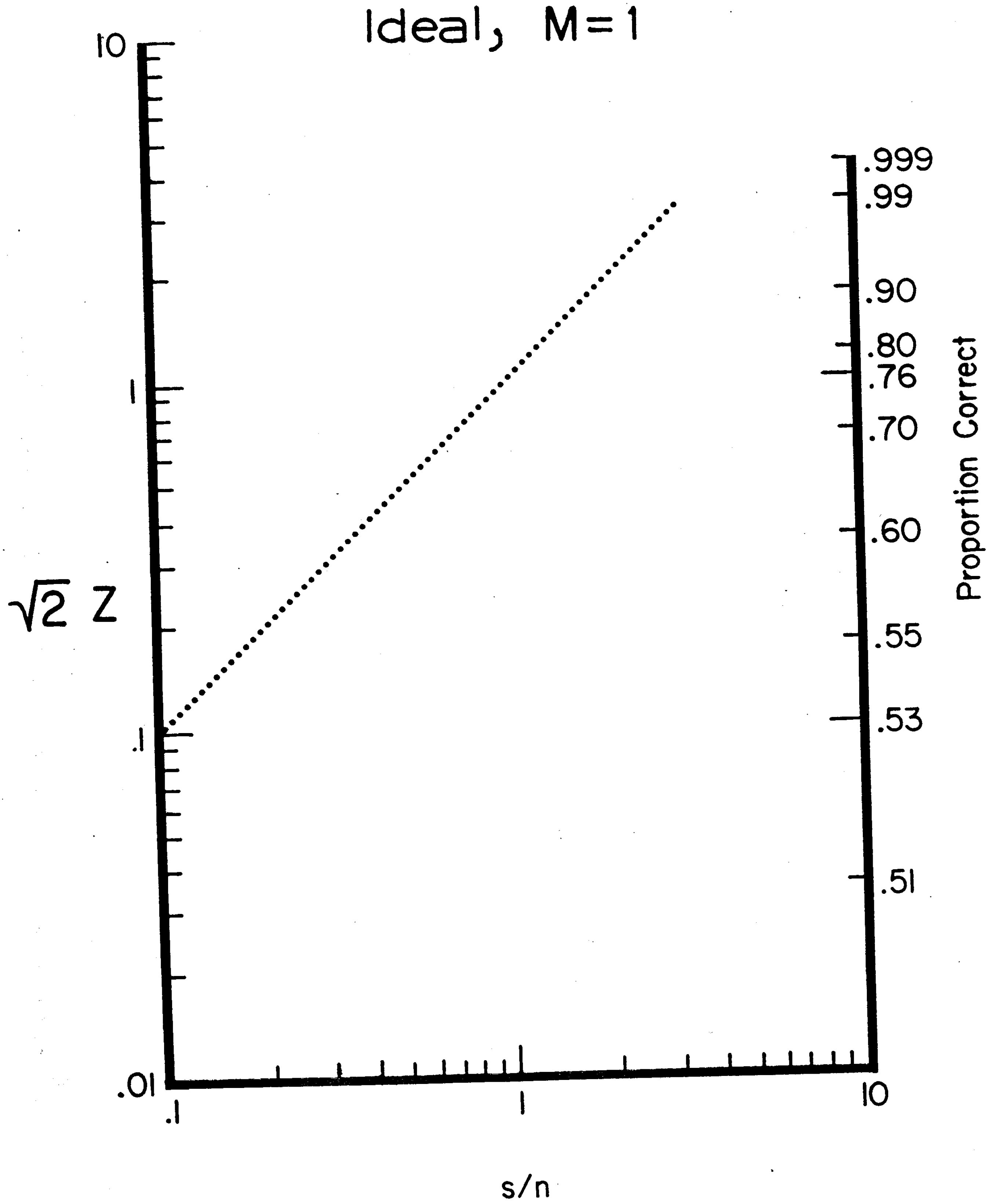
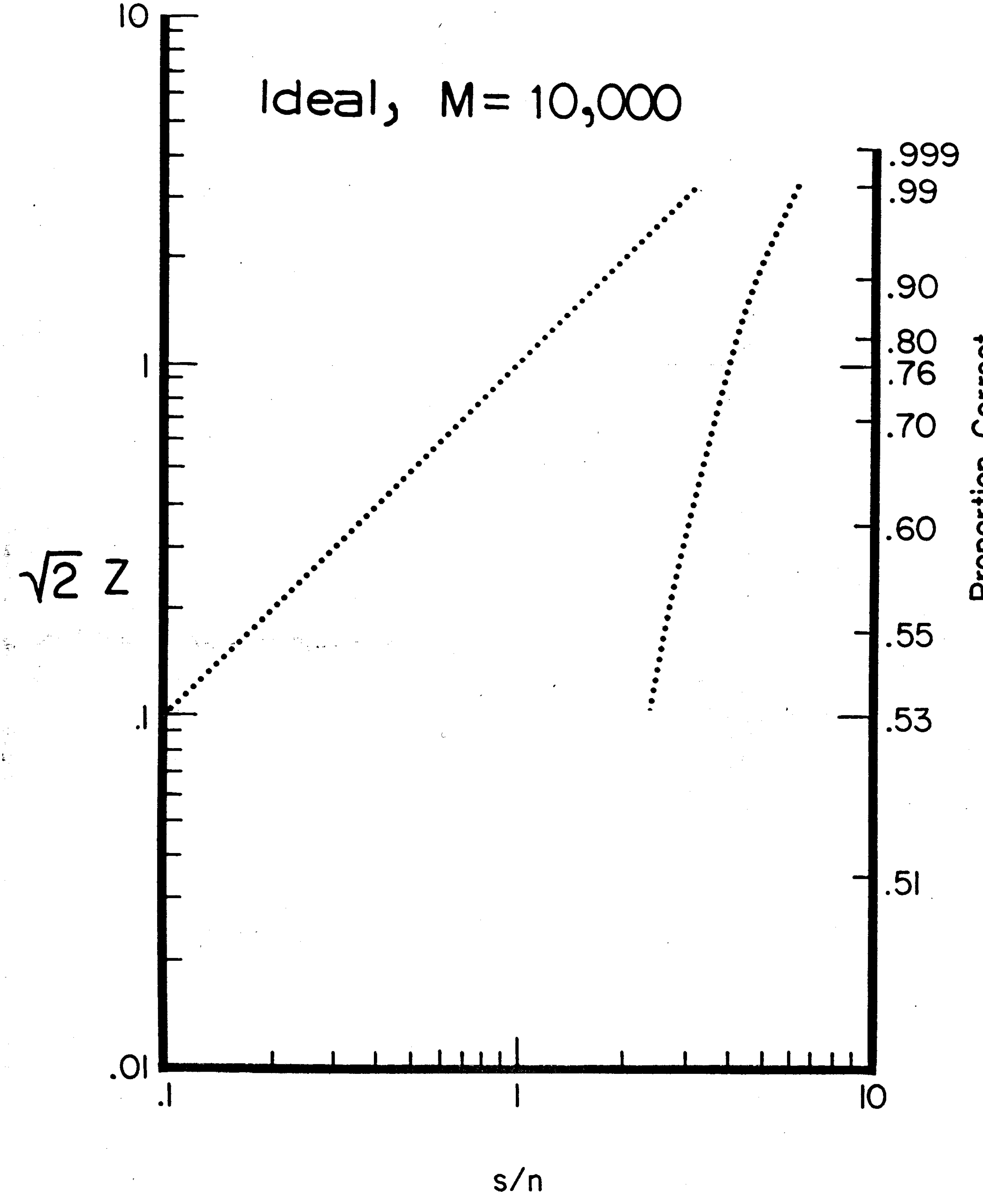
[12] Again, the ordinate is $\sqrt{2}$ times the normal deviate corresponding to the proportion correct, which Green and Swets (1966) would call "d'", but not Tanner and Birdsall (1958).


Ideal, M=1
10
1
.1
.01
√2 Z
.999
.99
.90
.80
.76
.70
.60
.55
.53
.51
Proportion Correct
.1
1
10
s/n


Ideal, M= 10,000
10
1
.1
.01
$\sqrt{2}$ Z
.1
1
10
s/n
.999
.99
.90
.80
.76
.70
.60
.55
.53
.51
Proportion Correct

performance for contrast discrimination, assuming the pedestal is highly detectable on its own.[14] The discrimination function has unity slope, just as without uncertainty. Uncertainty has very little effect on ideal discrimination performance if the pedestal is highly detectable on its own (see e.g. Lasely and Cohn 1981).

Figure 6: DP and Ideal

This last slide compares the observer's performance with and without uncertainty. The solid curves show the observer's psychometric function for the no-uncertainty condition. The dashed curves show his psychometric functions for 9,700-fold uncertainty. The uncertainty raised threshold by a factor of 1.5 for detection and by about 1.2 for discrimination. The small effect of uncertainty on threshold for detection and the even smaller effect of uncertainty on discrimination is consistent with the hypothesis of high intrinsic uncertainty.

The dotted lines are the ideal psychometric functions for 10,000-fold uncertainty. The constant of proportionality between signal-to-noise ratio and contrast was arbitrarily adjusted to slide the two ideal psychometric functions horizontally to the observer's psychometric functions for the no-uncertainty condition. The observer's psychometric functions without uncertainty are

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[13] Peterson, Birdsall, and Fox (1954), Nolte and Jaarsma (1967), and Pelli (1980)

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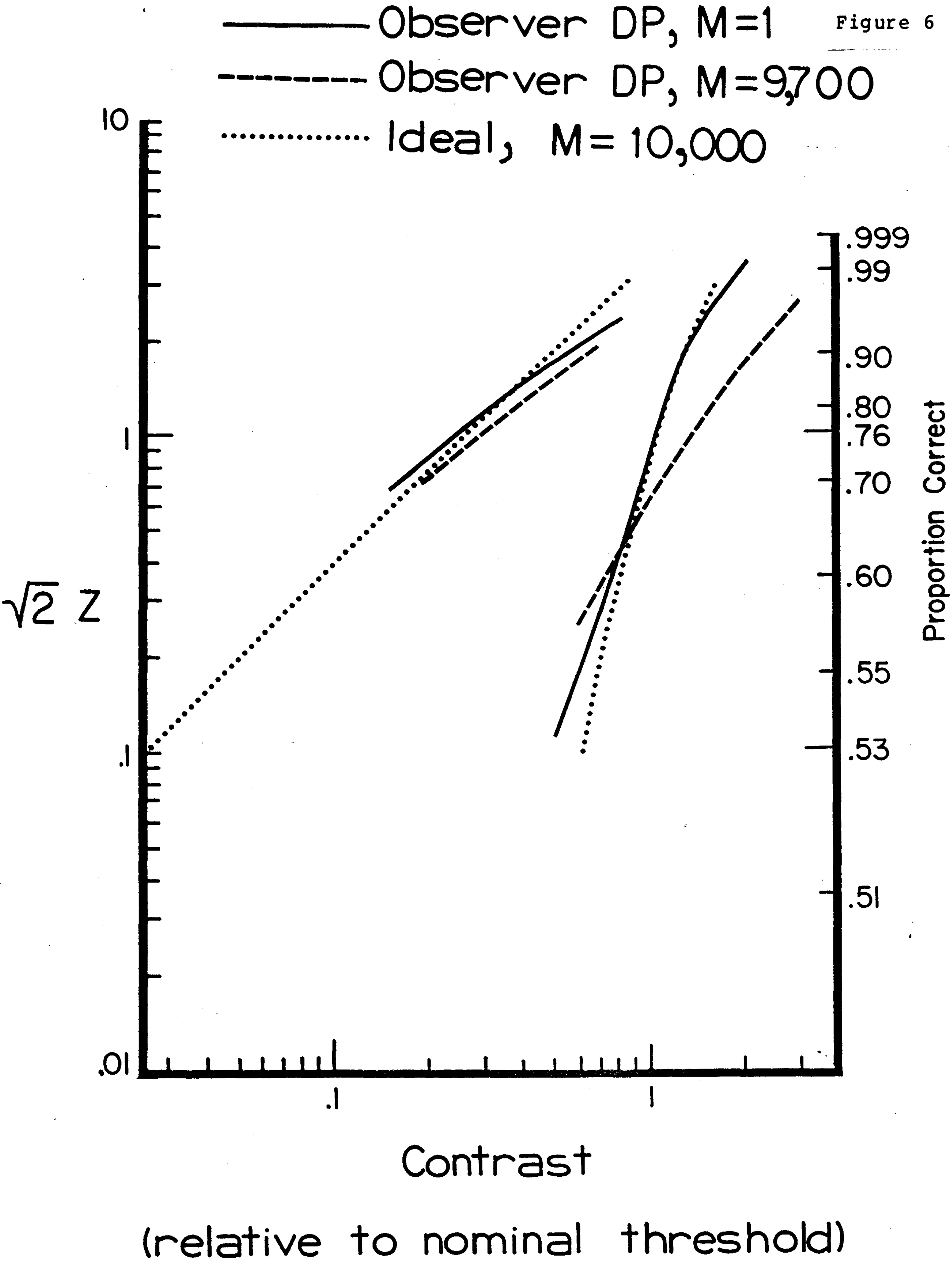
[14] The calculation assumed that the pedestal contrast had a detectability of d'=1, i.e. $\sqrt{2}z=1$.

Figure 6



Observer DP, M=1
Observer DP, M=9,700
Ideal, M= 10,000
10
1
.1
.01
$\sqrt{2}$ Z
.999
.99
.90
.80
.76
.70
.60
.55
.53
.51
Proportion Correct
.1
1
Contrast
(relative to nominal threshold)

strikingly similar to the ideal psychometric functions for ten-thousand-fold uncertainty. Similar detection results have been obtained from a second observer.

Thus, when the signal could appear at any one of nearly ten-thousand times and places, the observer's threshold was a factor of 1.5 times higher than when he was informed when and where the signal would appear. This rejects the high-threshold hypothesis which predicted <u>no</u> effect of uncertainty, and discredits the cross-correlation hypothesis which predicted a large effect.

With or without experimentally introduced uncertainty the human observer behaves very much like the ideal observer of one of many possible signals. It would seem that the observer has a high intrinsic uncertainty for detection of thin brief bars.

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